

*A Method To Measure The Focusing Properties  
(R<sub>-</sub>Matrix) of a Magnet*

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**A Method to Measure the Focusing Properties (R Matrix) of a Magnet\***N. Tsoupas, P. Thieberger, K. Brown, D. Gassner, W. Glenn, L. Ahrens, T. Roser,  
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**Abstract**

In this paper we discuss, and study the feasibility of a method that may be used to measure the focusing properties of a magnet. This method may prove valuable when applied to "non\_conventional" magnets that deviate from the usual dipole magnets or other multipole magnets (quadrupoles /sextupoles etc.) which are commonly used in a synchrotron or a beam line. In this category of "non\_conventional" magnets, fall special magnets, which come under the name "Snakes", which are being used in synchrotron accelerators[1,2,3,4] to introduce artificial spin resonances to help overcome the intrinsic and/or imperfection spin resonances which appear during the acceleration of polarized beams. This method of measuring the focusing properties of a magnet requires the use of "low energy" and "high rigidity" heavy-ions which may be obtained from the BNL Tandem accelerator.

In brief the method consists on, injecting "narrow beamlets" of heavy ions into a magnet and measuring the coordinates, of these "narrow beamlets", at the entrance and exit of the magnet.

From the measurement of the coordinates of the "narrow beamlets" we can deduce information on the R matrix (first order transfer matrix elements) and higher order matrix elements that define the focusing properties of the magnet.

**INTRODUCTION**

The common method, which is applied in the magnet division of the Brookhaven National Laboratory (BNL), to determine the focusing properties of a magnet, is to measure the magnetic field of the magnet by using various techniques[5] which provide accurate measurements to determine the focusing properties of a magnet like dipoles or higher order multipoles. The magnetic-field measurement method however is not very accurate when applied to measure the focusing

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properties of a magnet, like a Siberian Snake[3,4]. In this paper we study the possibility

to measure the focusing properties of a Helical Snake[3,4] or of any other magnet that does not lend itself to any of the magnetic measurements technique. In the rest of the paper we discuss the principle of the proposed method the apparatus for the required measurements, and the required accuracy of the measurements.

**PRINCIPLE OF THE METHOD**

The focusing properties of a magnet are considered known when the coordinates of a particle at the exit of a magnet can be determined, assuming that the coordinates of the particle at the entrance of the magnet are known. The above sentence can be expressed either, schematically in Figure 1, which shows a magnet with the coordinates of a particle at the entrance and exit coordinate systems, or mathematically in equation (1) below.

$$x_i(\text{out}) = \sum R_{ij} x_j(\text{in}) + \sum \sum W_{ikl}(\text{in}) x_k(\text{in}) x_l(\text{in}) + \sum \sum T_{imno} x_m(\text{in}) x_n(\text{in}) x_o(\text{in}) + \dots \text{HOT} \quad (1)$$

The notation in equation (1) is:

$(x_1, x_2, x_3, x_4, x_5, x_6) \Leftrightarrow (x, x', y, y', dl, \delta p/p)$   
Where  $x, y, x', y'$  are the lateral ( $x, y$ ) and angular ( $x', y'$ ) deviations of the particle from the trajectory of the central particle. The quantity  $\delta p$  is the momentum deviation of the particle's momentum  $p$  from the momentum  $p_0$  of the central-particle, and  $dl$  is the path length difference of the particle's path from the path of the central particle.

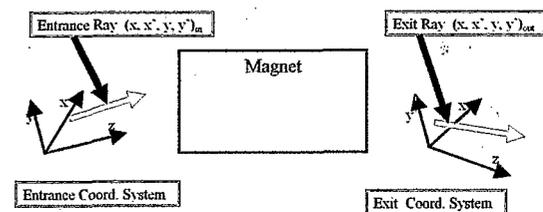


Figure 1. Schematic diagram of the magnet with the entrance end exit coordinate system. The beginning of the red arrows show the location of the particle and the direction of the arrow shows the direction of the particle at the entrance and exit coordinate systems.

The coefficients in equation (1) are defined as the partial derivatives of the output coordinates with respect to the input coordinates.

$$R_{ij} = \partial x_i(\text{out}) / \partial x_j(\text{in}) \quad 1^{\text{st}} \text{ order}$$

$$W_{ikl} = \partial x_i(\text{out}) / \{ \partial x_k(\text{in}) \partial x_l(\text{in}) \} \quad 2^{\text{nd}} \text{ order}$$

$$T_{imno} = \partial x_i(\text{out}) / \{ \partial x_m(\text{in}) \partial x_n(\text{in}) \partial x_o(\text{in}) \} \quad 3^{\text{rd}} \text{ order}$$

$$\dots\dots\dots$$

$$T_{imno\dots p} = \partial x_i / (\partial x_m \partial x_n \partial x_o \dots \partial x_p) \quad n^{\text{th}} \text{ order}$$

The knowledge of the coefficients  $R_{ij}$ ,  $W_{ijk}$ ,  $T_{imno}$ , and of the higher order coefficients, determine the focusing properties of any magnet.

In the "paraxial ray approximation", which assumes, that the momentum deviation  $\delta p$  of the momentum  $p$  of the particle is much smaller than momentum  $p_0$  ( $\delta p/p_0 \leq 0.01$ ) of the central particle, and that the lateral coordinates  $x, y$  in both entrance and exit coordinate systems are much smaller than the radius of curvature<sup>1</sup>  $\rho$ , of the orbit of a particle moving in the magnetic field  $B$  of the magnet, most of the higher than the 1<sup>st</sup> order terms which appear in equation (1) ( $W_{ijk}$ ,  $T_{imno}$  ....etc) are usually negligible.

Therefore, the equation (1) above can be written

$$x_i(\text{out}) = \sum R_{ij} x_j(\text{in}) + \text{"few HOT"}. \quad (2)$$

In equation (2) the expression "few HOT" includes the higher order term which are comparable in magnitude to the first order terms. In summary, the first order coefficients  $R_{ij}$ , and the "most important" higher order coefficients appearing in equations (1,2) can be computed by measuring the coordinates  $(x, x', y, y')_{\text{exit}}$  of an "adequate" number of particles at the exit of the magnet and the corresponding coordinates  $(x, x', y, y')_{\text{in}}$  of the particles at the entrance coordinate system. By solving equation (2) we can determine the first order  $R_{ij}$  terms as well as the "most important" higher order coefficients.

## DESCRIPTION OF THE APPARATUS

A schematic diagram of the proposed apparatus which can be used to measure the focusing properties of a magnet is shown in figure 2.

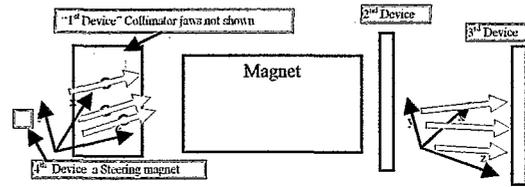


Figure 2. Schematic diagram of the magnet with the various devices which define the position  $(x, y)$  and direction  $(x', y')$  of the rays at the entrance and exit of the magnet.

The apparatus which schematically shown in Figure 2 consist of four separate devices.

- The first device is the "1<sup>st</sup> ion-position defining device" which is placed at the entrance of the magnet and is located at a specified distance from the magnet iron, just before the start of the fringe field of the magnet. The "1<sup>st</sup> ion-position defining device" consist of a metal plate, labeled as "collimator" in Figure 2, which is thick enough to stop the incident ions. The collimator plate has "pin holes"<sup>2</sup> through which the ions will pass to enter the magnet. The location of the center of the pin holes defines the location  $(x, y)_{\text{in}}$  of the particles at the entrance. Two sets of "jaws", one horizontal and one vertical which can move parallel to the surface of the "collimator" will allow to select one pin hole at a time that the ions will enter the magnet. If the "jaw system" has an additional degree of freedom which allows the "jaw system" to rotate about the longitudinal axis ( $z$ -axis), this will permit the selection of a set of pin-holes which are lined up along any direction on the collimator plate. The "jaws" should be thick enough to stop the particles entering the magnet.
- The second device is the "2<sup>nd</sup> ion-position measuring device" which is located at a specified distance, just after the fringe field of the magnet, consists of a light emitting foil (visual Flag) and a CCD camera connected to a computer. The light emitting foil is a compound of Gadolinium Oxy-Sulfide doped with Terbium ( $Gd_2O_2S:Tb$ ). The compound is bonded on an Al foil. The ions emerging from the pin holes of the "1<sup>st</sup> ion-position defining device" will generate light on the foils of the 2<sup>nd</sup> or 3<sup>rd</sup> measuring devices. The emitted light will be detected by the CCD camera and the computer will calculate the

<sup>1</sup> The radius of curvature  $\rho$  is defined in the equation  $B \cdot \rho = k \cdot p / q$  ( $B$  is the field of the magnet,  $p$  is the momentum, and  $q$  is the charge state of the ion,  $k$  is a constant which depends on the units),

<sup>2</sup> The size of the holes depends on the magnet to be measured. For the Partial Snake magnet the diameter of the holes should be no greater than 0.5 mm.

location of each “pin-hole image” on the visual flags (foils).

A schematic diagram of the detection system is shown in Fig. 3. An alternative ion position measuring system which utilizes thin wires is discussed in ref. [5]. The direction of the rays  $(x',y')_{in}$  at the entrance of the magnet can be determined from the coordinates  $(x,y)_{in}$  of the rays at the entrance, as defined by the “1<sup>st</sup> ion-position defining device”, and the coordinates  $(x,y)_{out}$  of the rays at the exit as measured by the “2<sup>nd</sup> ion-position measuring device” when the field of the magnet is off.

- The third device “3<sup>rd</sup> ion-position measuring device” is located at the exit of the magnet at a specified distance, from the location of “2<sup>nd</sup> ion-position measuring device” otherwise it is identical to the “2<sup>nd</sup> ion-position measuring device”. Position measurements of the ions taken from the 2<sup>nd</sup> and 3<sup>rd</sup> “ion-position measuring devices” will determine the direction of the ions at the exit of the magnet  $(x',y')_{out}$ .
- The fourth device is the “ray-direction defining” magnet which is a dipole magnet that can change the direction  $(x',y')_{in}$  of the rays at the entrance of the magnet and also serves to select the charge-state to be used when using heavy ions.

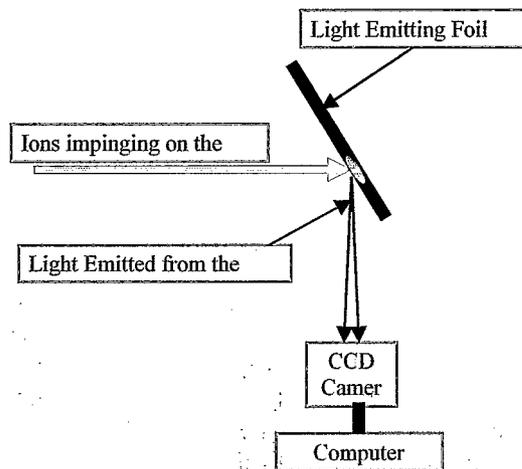


Figure 3. Schematic diagram of an ion-position measuring system to be used in the position measurement of the pencil-like ion beams which will enter and exit the magnet.

### COMPUTER SIMULATIONS

The method to measure the focusing properties of a magnet was tested using computer simulations on a 3D model of the warm helical

snake [3] which is installed in AGS. The raytracing into the magnet was performed with the computer code SPRAY [6]. The required 3D magnetic fields of the magnet were computed using the computer code opera [7] on a 3D model of the magnet [4]. Error analysis of the proposed method as well as comparison of the method with the magnetic measurements method has been performed and appears in [4].

### SUMMARY

A method to determine experimentally the focusing properties of a magnet has been discussed. This method determines the first order matrix elements ( $R_{matrix}$ ) of a magnet and the most “significant” higher order matrix elements. The error in the determination of the first order matrix elements is “reasonably small” for the measured  $R_{matrix}$  to be used in various calculations which may involve the focusing properties of the measured magnet.

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