STOCHASTIC COOLING POWER REQUIREMENTS *

J. Wei†, M. Blaskiewicz, M. Brennan, Brookhaven National Laboratory, USA

Abstract

A practical obstacle for stochastic cooling in high-energy colliders like RHIC is the large amount of power needed for the cooling system. Based on the coasting-beam Fokker-Planck (F-P) equation, we analytically derived the optimum cooling rate and cooling power for a beam of uniform distribution and a cooling system of linear gain function. The results indicate that the usual back-of-envelope formula over-estimated the cooling power by a factor of the mixing factor M. On the other hand, the scaling laws derived from the coasting-beam Fokker-Planck approach agree with those derived from the bunched-beam Fokker-Planck approach if the peak beam intensity is used as the effective coasting-beam intensity. A longitudinal stochastic cooling system of 4 – 8 GHz bandwidth in RHIC can effectively counteract intrabeam scattering, preventing the beam from escaping the RF bucket becoming debunched around the ring.

1 INTRODUCTION

Stochastic cooling [1, 2] has long been recognized as a viable approach to counteract the emittance growth and beam loss caused by intrabeam scattering in RHIC [3, 4]. Theoretically, with a transverse cooling system of frequency bandwidth from 4 to 8 GHz, the (normalized 95%) emittance of a gold beam of 10^9 particles per bunch can be preserved at 30 $\pi\mu$m. With a longitudinal cooling system of the same frequency bandwidth, the beam escaping from the RF bucket debunched around the ring can be eliminated [5].

A possible technical obstacle is the existence of very strong coherent components at GHz frequency range that would saturate the electronics of the cooling system and swamp the true stochastic information. Due to this problem, attempts at implementing bunched-beam stochastic cooling at the Tevatron and the SPS were unsuccessful. On the other hand, cooling of the heavy ion beam in RHIC has the advantage that the signal-to-noise ratio is high due to the high charge state, and that longitudinally the beam occupies a large fraction of the RF bucket. Recent measurements of Schottky signals of the gold beam indicate that stochastic cooling in RHIC would not be impeded by anomalous coherent components in the Schottky signals [6, 7].

Practically, the obstacle for stochastic cooling in RHIC is the large amount of power needed for the cooling system [3]. Early study using the Fokker-Planck approach indicated that the power needed is proportional to the energy spread of the beam to the forth power [4]. With a total kicker coupling-resistance of 6.4 kΩ, the power needed for longitudinal cooling at beam storage is several kilo Watts at a frequency range from 4 to 8 GHz. However, a comparison between the Fokker-Planck calculation [4] and the estimate given in Ref. [3] indicates a difference in the scaling behavior of the cooling power when the mixing factor [2] is larger than unity. The purpose of this note is to present the analytical derivation of the cooling power scaling law and to discuss applications in RHIC.

2 LOPNGITUDINAL F-P EQUATION

Assume that the evolution of the beam distribution is slow during a synchrotron-oscillation period. The evolution of the longitudinal density function $\Psi(W)$ can be described by the Fokker-Planck equation [8, 9, 2]

$$\frac{\partial \Psi_L}{\partial t} = -\frac{\partial}{\partial W}\left(F_L\Psi_L\right) + \frac{1}{2}\frac{\partial}{\partial W}\left(D_L\frac{\partial \Psi_L}{\partial W}\right). \quad (1)$$

with the boundary condition

$$\begin{cases} -F_L\Psi_L + \dfrac{1}{2}D_L\dfrac{\partial \Psi_L}{\partial W} = 0 & W = 0 \\ \Psi = 0 & W = W_{max} \end{cases} \quad (2)$$

where $W \equiv \frac{\Delta E}{\omega_s}$ is the scaled energy deviation, ω_s is the revolution frequency. Neglect the thermal noise which is small compared with the Schottky noise for heavy-ion beams. The drifting coefficient F and the diffusion coefficients D_L are given by

$$\begin{aligned} F_L(W) = & \lim_{\Delta t\to 0}\frac{1}{\Delta t}\langle\int_0^{\Delta t} dt\, U_W(t)\rangle \\ D_L(W) = & \lim_{\Delta t\to 0}\frac{1}{\Delta t}\langle\int_0^{\Delta t} dt \int_0^{\Delta t} dt'\, U_W(t)U_W(t')\rangle \end{aligned} \quad (3)$$

where

$$U_{W,i} = \frac{Ze}{\omega_s}\sum_{n=-\infty}^{\infty} V^K(t)\,\delta\left(t - \frac{2\pi n}{\omega_i} - \frac{\phi_i}{\omega_s} - \frac{\theta^K}{\omega_s}\right), \quad (4)$$

$$\begin{aligned} V^K(t) = & \frac{Ze}{2\pi}\int_{-\infty}^{\infty} d\omega e^{i\omega t}G_L(\omega)\sum_{j=1}^{N}\sum_{m=-\infty}^{\infty} \\ & \exp\left[-i\omega\left(\frac{2\pi m}{\omega_j} + \frac{\phi_j}{\omega_s} + \frac{\theta^P}{\omega_s}\right)\right] \end{aligned} \quad (5)$$

with ϕ_i the phase of the test particle, θ^K the azimuthal location of the kicker in the ring, the superscripts K and P indicating the kicker and the pick-up, and $G_L(\omega)$ is the gain function. The drifting and diffusion coefficients become

$$F_L = \frac{z^2e^2\omega_i}{4\pi^2}\sum_{m=-\infty}^{\infty} G_L(m\omega_i)e^{-im(\theta^P-\theta^K)\frac{\Delta\omega_i}{\omega_s}} \quad (6)$$

* Work performed under the auspices of the U.S. Department of Energy.

† jwei@bnl.gov

$$D_L = \frac{z^4 e^4 \omega_i^2}{8\pi^3} \sum_{j=1}^{N} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{|G_L(m\omega_j)|^2}{|m|} \rho(\omega_j)|_{\omega_j=\frac{n}{m}\omega_i} \quad (7)$$

where the factor $e^{-im(\theta^P-\theta^K)\frac{\Delta\omega_i}{\omega_s}}$ represents the bad mixing between the pick-up and the kicker, and the summation is over the effective frequency range of the cooling system. The average power required for cooling is

$$\bar{P}_L = \lim_{\Delta t\to 0} \frac{1}{\Delta t} \int_0^{\Delta t} dt \, \frac{\langle (V^K(t))^2 \rangle}{n^K R^K} \quad (8)$$

where n^K is the number of kicker pairs, and R^K is the coupling resistance of each kicker pair.

3 TRANSVERSE F-P EQUATION

Evolution of the transverse density function $\Psi(I)$ is described by the Fokker-Planck equation [8, 9, 2]

$$\frac{\partial \Psi_T}{\partial t} = -\frac{\partial}{\partial I}(F_T \Psi_T) + \frac{1}{2}\frac{\partial}{\partial I}\left(D_T \frac{\partial \Psi_T}{\partial I}\right). \quad (9)$$

with

$$\begin{cases} -F_T\Psi_T + \frac{1}{2} D_T \frac{\partial \Psi_T}{\partial I} = 0 & I = 0 \\ \Psi = 0 & I = I_{max} \end{cases} \quad (10)$$

where I is the transverse action,

$$\begin{aligned} F_T(I) &= \lim_{\Delta t\to 0} \frac{1}{\Delta t} \langle \int_0^{\Delta t} dt \, U_I(t) \rangle \\ D_T(I) &= \lim_{\Delta t\to 0} \frac{1}{\Delta t} \langle \int_0^{\Delta t} dt \int_0^{\Delta t} dt' \, U_I(t) U_I(t') \rangle \end{aligned} \quad (11)$$

with

$$\begin{aligned} U_{I,i} &= -\sqrt{2 I_i \beta_x^K} \sin\phi_{\beta,i} \sum_{n=-\infty}^{\infty} \frac{U^K(t)}{\omega_s} \\ &\quad \delta\left(t - \frac{2\pi n}{\omega_i} - \frac{\phi_i}{\omega_s} - \frac{\theta^K}{\omega_s}\right) \end{aligned} \quad (12)$$

$$\begin{aligned} U^K(t) &= \frac{Z^2 e^2}{2\pi} \int_{-\infty}^{\infty} d\omega e^{(i\omega t} G_T(\omega) \sum_{j=1}^{N} \sqrt{2 I_j \beta_x^P} \\ &\quad \sum_{m=-\infty}^{\infty} \cos\phi_{\beta,j} e^{-i\omega\left(\frac{2\pi m}{\omega_j} + \frac{\phi_j}{\omega_s} + \frac{\theta^P}{\omega_s}\right)} \end{aligned} \quad (13)$$

The drifting coefficient becomes Q_x is the fractional transverse tune, and The factor $e^{-i(m\mp Q_x)(\theta^P-\theta^K)\frac{\Delta\omega_i}{\omega_s}}$ is the "bad mixing" effect from the delay between the pick-up and the kicker. Assume that the gain G_T is the same at the upper and lower betatron sidebands, and absorb the factor $e^{-i(m\mp Q_x)(\theta^P-\theta^K)}$ into the gain function. We have

$$\begin{aligned} F_T &\approx \frac{z^2 e^2 \sqrt{\beta_x^K \beta_x^P} \omega_i}{4\pi^2} \sum_{m=-\infty}^{\infty} G_T\left[(m \mp Q_x)\omega_i\right] \\ &\quad e^{-im(\theta^P-\theta^K)\frac{\Delta\omega_i}{\omega_s}} \sin\left[Q_x(\theta^P - \theta^K)\right] I \end{aligned} \quad (14)$$

where the factor $\sin\left[Q_x(\theta^P - \theta^K)\right]$ indicates that a betatron phase advance of $\pi/2$ between the pick-ups and the kickers optimizes the performance, and the summation is over the effective frequency range of the cooling system. The diffusion coefficient becomes

$$\begin{aligned} D_T &= \frac{z^4 e^4 \omega_i^2 \beta_x^K \beta_x^P I}{8\pi^3} \sum_{j=1}^{N} I_j \\ &\quad \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{|G_T[(m \mp Q_x)\omega_j]|^2}{|m|} \rho(\omega_j)|_{\omega_j=\frac{n}{m}\omega_i} \end{aligned} \quad (15)$$

where the summation over j indicates the contribution from all the particles, and the double summation over m and n considers the case of frequency overlapping. The average power required for cooling is

$$\bar{P} = \lim_{\Delta t\to 0} \frac{1}{\Delta t} \int_0^{\Delta t} dt \, \frac{\langle V_k^2(t) \rangle}{n_k R_k}, \quad V^K(t) = \frac{2\pi\beta c p_s \Delta_x^K}{Z e L^K} U^K(t) \quad (16)$$

where n_k is the number of kicker modules, R_k and L^K are the coupling resistance and length of each kicker module, respectively, $2\Delta_x^K$ is the kicker gap size, and $p_s = A m_0 \beta c \gamma$ is the synchronous momentum.

4 COOLING RATE AND POWER

Longitudinally, the cooling rate of the beam energy spread $\langle W \rangle = 2\int_0^{W_{max}} W \, \Psi_L(W) \, dW$ is given by

$$\frac{\partial \langle W \rangle}{\partial t} = 2 \int_0^{W_{max}} dW \left(F_L + \frac{1}{2}\frac{\partial D_L}{\partial W}\right) \Psi_L(W) \quad (17)$$

Assume a linear gain function "notched" at multiples of the revolution frequency, $G_L(m\omega) = gmW$, $\Delta\omega = \omega - m\omega_s = -\frac{\eta\omega_s^2}{E_s\gamma^2} W$ where $m\omega_s$ is the nearest multiple of the revolution frequency to ω, $\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$ is the slip factor, $E_s = A m_0 c^2 \gamma$ is the beam energy, and γ_t is the transition energy. Denote the effective frequency range of cooling from $n_1\omega_s$ to $n_2\omega_s$, $\Delta n = n_2 - n_1$, $\bar{n} = \frac{n_1 + n_2}{2}$, and the Schottky bands are assumed to be non-overlapping. For a uniform density distribution

$$\rho[\omega(W)] = \begin{cases} \frac{N}{2\Delta\omega_s} & |W| < W_0 \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

$\Delta\omega_s$ is the frequency spread. neglecting the effect of "bad mixing", the maximum cooling rate that corresponds to the minimum cooling time τ_{min} is

$$\tau_{L,min}^{-1} \approx -\frac{\Delta n \omega_s}{2\pi N M} \quad (19)$$

The mixing factor M is given by

$$M = \frac{1}{\bar{n}|\eta|(\Delta\hat{p}/p)_0} = \frac{1}{\sqrt{3}\bar{n}|\eta|\sigma_p}, \quad \text{for } M > 1 \quad (20)$$

where σ_p is the rms spread in momentum ($\Delta p/p$). The average power needed for longitudinal cooling is

$$\bar{P}_L \approx \frac{2}{f_s n^K R^K} \frac{1}{\tau_{min} M} \left(\frac{\beta^2 E_s \sigma_p}{Ze}\right)^2 \tag{21}$$

where $f_s = \omega_s/2\pi$ is the revolution frequency.

Tansversely, the cooling rate of action $\langle I \rangle = \int_0^{I_{max}} I\,\Psi_T(I)\,dI$ is given by

$$\frac{\partial \langle I \rangle}{\partial t} = \int_0^{I_{max}} dI \left(F_T + \frac{1}{2}\frac{\partial D_T}{\partial I}\right) \Psi_T(I) \tag{22}$$

Assume constant gain function at the betatron sidebands of the multiples of the revolution frequency, $G_T[(m \mp Q_x)\omega] = g$. The maximum cooling rate that corresponds to the minimum cooling time $\tau_{T,min}$ is

$$\tau_{T,min}^{-1} = \frac{1}{\langle I \rangle} \frac{\partial \langle I \rangle}{\partial t}|_{max} \approx -\frac{\Delta n \omega_s}{\pi N M} \tag{23}$$

The average power needed for the transverse cooling is

$$\bar{P}_T \approx \frac{2\langle \epsilon_x \rangle \Delta_x^2}{f_s n_k R_k \beta_x^K} \frac{1}{\tau_{min} M} \left(\frac{\beta^2 E_s}{ZeL^K}\right)^2 \tag{24}$$

where $\langle \epsilon_x \rangle = 2\langle I \rangle$ is the unnormalized average emittance, and $2\Delta_x$ is the kicker gap height.

5 RHIC EXAMPLE

Consider longitudinal and transverse stochastic cooling of a gold beam at RHIC storage. As shown in Table 1, the beam grows under intrabeam scattering during a typical 10-hour store [10]. Due to the growth in momentum spread, the mixing factor M varies from 9.4 to 5.6. The optimum cooling time varies from 8.7 to 3.2 hours for the momentum spread, and from 4.4 to 1.6 hours for the transverse emittance. With 128 pairs of kickers each at 50 Ω coupling resistance, the average power for longitudinal cooling varies from 0.15 kW to 2.0 kW. Again with 128 pairs of 50 Ω kickers, each of effective length 1 cm, arranged at a gap height of $2\Delta_x = 4$ cm at a location of $\beta_x^K = 20$ m, the average power for transverse cooling varies from 10 W to 114 W.

Table 1: Parameter example for stochastic cooling of gold beam at RHIC storage.

Mass number, A	197	
Change state, Z	79	
Energy per nucleon, E_s/A	100	GeV/u
Revolution frequency, $f_s = \omega_s/2\pi$	78	kHz
Bunch intensity	1	10^9
Momentum slip factor, $\|\eta\|$	1.9	10^{-3}
RF voltage	6	MV
RF harmonic, h	2520	
Bunch length rms (begin - end)	0.11 - 0.19	m
Bunch length rms (begin - end)	27° - 45°	
Bunching factor (begin - end)	0.19 - 0.31	
Eff. bunch intensity (begin - end)	1.33 - 0.81	10^{13}
Momentum spread rms (begin - end)	0.44 - 0.71	10^{-3}
Tran. norm. 95% emittance	15 - 40	μm
Cooling bandwidth	4 - 8	GHz
Mixing factor, M (begin - end)	9.4 - 5.6	
Momentum cooling time	8.7 - 3.2	hour
Emit. cooling time	4.4 - 1.6	hour

6 DISCUSSIONS AND SUMMARY

Based on the coasting-beam Fokker-Planck equation, we analytically derived the optimum rate and power for the longitudinal stochastic cooling of a beam of uniform distribution and a linear gain function. The results indicate that the usual back-of-envelope formula [3] over-estimated the cooling power by a factor of the mixing factor M. On the other hand, the scaling laws derived from the coasting-beam Fokker-Planck approach agree with those derived from the bunched-beam Fokker-Planck approach [4] if the peak beam intensity is used as the effective coasting-beam intensity. Although we have ignored signal suppression for the entire discussion, the conclusion holds.

A longitudinal stochastic cooling system of 4 – 8 GHz bandwidth in RHIC can effectively counteract intrabeam scattering, preventing the gold beam from escaping the RF bucket becoming debunched around the ring. Combining with a transverse stochastic cooling system of the same frequency bandwidth to contain the transverse emittance, we expect a significant increase in the average luminosity.

We would like to thank J. Marriner and D. Moehl for many useful discussions.

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