BNL preprint BNL-71490-2003-CP
Aug. 2003

Analysis of space charge driven modulation in electron bunch energy spectra

Timur Shaftan and Li Hua Yu
NSLS-BNL, Upton, NY, 11973

Abstract

As was discussed earlier [1,2] longitudinal space charge force in initially nonuniform bunch transforms density fluctuations into energy modulation along the bunch. For characterization of the resulted energy modulation one can chirp the bunch using accelerator section, located upstream of beam spectrometer, and record energy spectrum of such chirped bunch. Measured spectrum shows structure with parameters, depending on the bunch properties. In this paper we present analysis of the structure in the bunch energy spectrum and its connection with energy modulation along the bunch.

Introduction

The Deep-Ultra Violet Free Electron Laser (DUV-FEL) is under development in the Source Development Laboratory (BNL). The goal of the project is to generate UV radiation at 100 nm wavelength and below. The accelerator [3] consists of a BNL/SLAC/UCLA type electron gun driven by a Ti:Sa laser, four SLAC-type linac tanks and a four magnet chicane. A 4.5 MeV electron beam leaves the gun, is accelerated up to 70 MeV in two linac sections, compressed in the magnetic chicane, and accelerated up to 140 MeV (200 MeV maximum) in the last two linac sections. The RF gun currently can produce 2 ps (RMS) bunches with 300 pC of charge and 2-4 mm-mrad normalized emittances. The peak current required for successful FEL operation is on the order of 350 Amp.

Several years ago new interesting effect was obtained in DUV-FEL linear accelerator. Energy spectrum of high-brightness electron bunch revealed spiky structure with sub-picosecond spike separation [4]. Beam studies demonstrated sensitivity of structure to the beam parameters [2].

It has been shown [1], that the space charge force for our experimental conditions can be strong enough to transform small longitudinal density non-uniformities into a modulation of the average energy along the bunch. Chirping such an energy-modulated bunch one would see sharp spikes in the energy spectrum, corresponding to variations of the energy. At the same time this effect cannot enhance the density bunching in the electron beam, since the longitudinal space charge force is equivalent to pure imaginary capacitive impedance.

The purpose of this paper is to characterize short-wavelength energy-time correlations in the bunch energy spectra. We discuss basic analysis of the measured spiky structure and it’s connection with energy modulation along the bunch.

Modulation analysis

Let us consider experimental set-up consisting of accelerating cavity and energy spectrometer. Following "zero-phasing" method [5,6], accelerating cavity imparts energy chirp in the bunch, without changing average beam energy. If initially there is no correlation between energy and time, energy spectrum of the chirped bunch (CBE spectrum) exactly represents longitudinal density distribution. In this case we can scale energy spectrum in the units of time in the beam rest frame. Therefore energy interval in CBE spectrum is proportional to the time interval:

$$\Delta t = \frac{1}{\omega_{RF} E_{ch}} \Delta E = \frac{1}{h} \Delta E, \tag{1}$$

where ω_{RF} and E_{ch} are RF frequency and energy gain of accelerating cavity. Parameter h is known as energy chirp. If we performed Fourier analysis of such CBE spectrum, resulted Fourier spectrum would contain all information about longitudinal phase space.

Let us consider more complicate case of a bunch with sinusoidal energy modulation. In order to find expression for the chirped bunch energy spectrum we assume model of coasting beam with peak current I_0, energy spread σ_E and chirp h. Modulation is characterized by frequency ω and amplitude Δ. Expression (2) gives distribution function for such beam in phase space variables time t and energy E.

$$f_0(t,E) = \frac{I_0}{\sqrt{2\pi}\sigma_E} \exp\left[-\frac{(E_1(t,E))^2}{2\sigma_E^2} \right], \text{ where } E_1(t,E) = E - ht - \Delta \sin(\omega t) \tag{2}$$

Using similar to [7] approach, we represent distribution function as following:

$$f(t,E_1) = \int f_0(t,E') \cdot \delta(E' - E_1) dE' \tag{3}$$

In order to find energy spectrum g(E) we have to integrate (3) over time:

$$g(E) = \int f_0(E') dE' \int \delta(E' - E + ht + \Delta \sin(\omega t)) dt \tag{4}$$

Remarkable property of the last integral in (4) is that it represents periodic function of energy for any values of parameters ω, h and Δ. Thus we can get this integral, using one of δ-function properties.

$$\int \delta(E' - E + ht + \Delta \sin(\omega t)) dt = \sum_i \frac{1}{1 + \frac{\omega\Delta}{h} \cos\left(\omega \cdot t_i(E'-E)\right)}, \text{ where } t_i(E'-E) \text{ are roots}$$

of the equation: $E' - E + ht_i + \omega\Delta \sin(\omega t_i) = 0$. The equation has at least one root due to continuity of the phase space. Special case of more than one root happens when $h/(\Delta\omega)>1$, i.e. when correlated energy change across a single modulation period is smaller than the amplitude of energy modulation. We will address this situation as "overmodulation".

Expanding (4) in Fourier series we finally get the expression for CBE spectrum.

$$g(E)=\frac{I_0}{h}\left[1+2\sum_{n=1}^{\infty}J_n\left(n\frac{\omega}{h}\Delta\right)\exp\left\{-\frac{1}{2}\left(n\frac{\omega}{h}\sigma_E\right)^2\right\}\cos\left(n\frac{\omega}{h}E\right)\right], \qquad (5)$$

Sum of the different roots has been taken into account in (5). As one can see from expression (6), spectrum contains a sequence of harmonics with heights, determined by the energy modulation amplitude, intrinsic energy spread and parameter $\frac{\omega}{h}$.

For comparison, an example of measured CBE spectrum is shown on Fig. 1. Asymmetric shape of the spectrum caused by RF nonlinearity. Fourier transform of the CBE spectrum contains a family of harmonics with fundamental harmonic located at 3 THz. Low frequency part of the spectrum (below 2 THz) is mainly contributed by the smooth bunch form-factor.

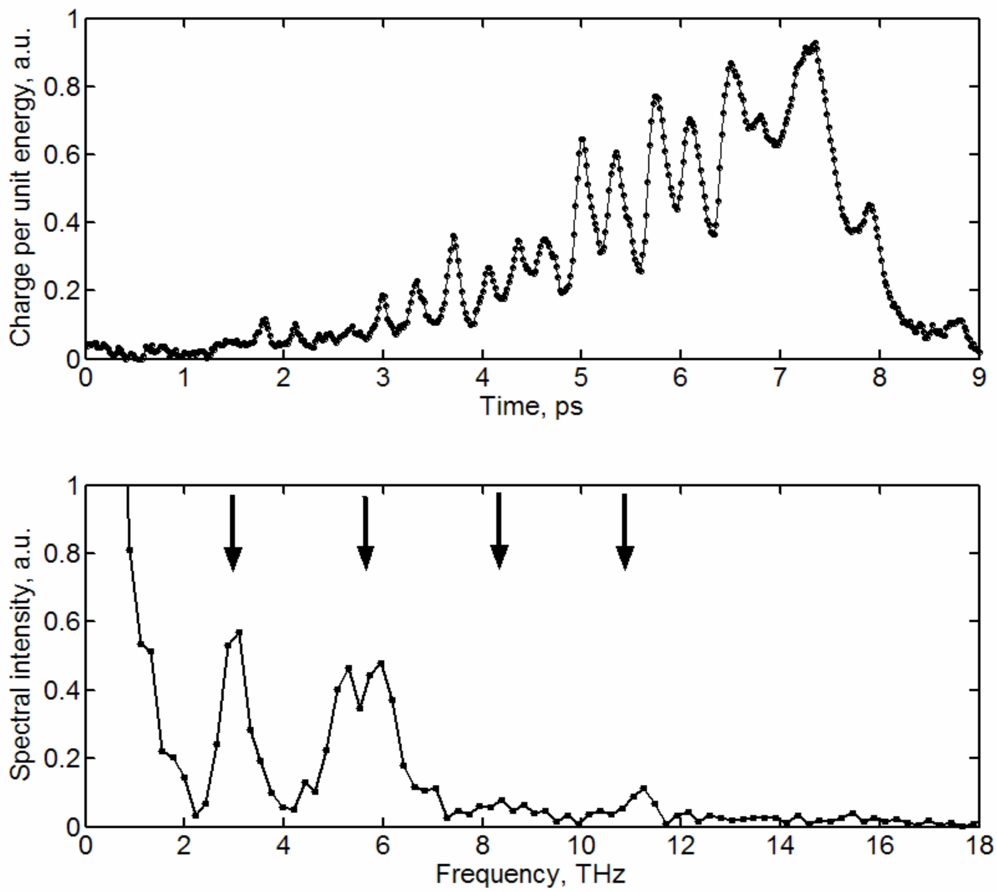

Fig. 1: Measured CBE spectrum and its Fourier transform. Arrows show locations of modulation harmonics.

In principle, energy modulation amplitude and intrinsic energy spread for this spectrum could be determined using expression (5). Serious limitations of the described analysis come from the assumed model of coasting beam with constant peak current. One of the problems arises from unknown bunch density distribution. Direct single-shot measurement of the bunch profile is nearly impossible in femtosecond range. Another problem is that density distribution must contain short-wavelength modulation at the same frequency range as energy modulation [1]. Therefore density modulation contributes in CBE spectrum. Following our analysis for DUV-FEL experimental set-up, relative fluctuations in the bunch longitudinal density are expected to be in the range of a few percent [2]. This makes the contribution of density modulation to the spiky structure in the bunch spectrum small enough.

In addition to previous complications, phase space of a real beam is usually dominated by various nonilinear terms, introduced by RF waveform and space charge. In this case energy chirp becomes time-dependent and the last integral in (4) is not a periodic function of time. An example of the chirped beam image is shown in Fig. 2. One of the interesting features here is evolution of the modulation wavelength along the bunch, corresponding to nonlinear chirp. "Overmodulated" periods can be clearly seen on the left side of the image. Every couple of double spikes in this region represents a single modulation period. Tail of the bunch is folded back over, introducing bright region on the right side of the image. This phenomenon is caused by space charge, creating time-energy correlation, proportional to the derivative of the bunch density distribution [9].

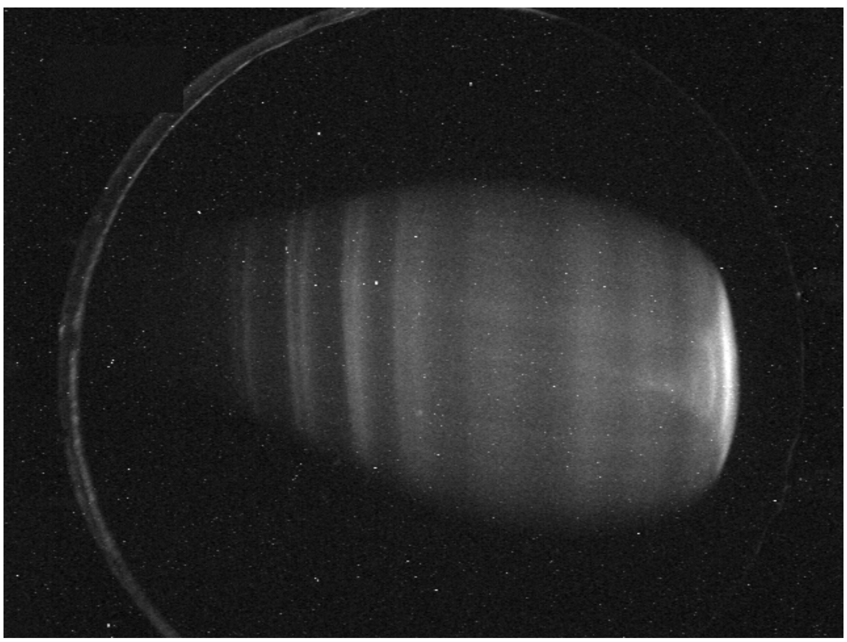

Fig. 2: Recorded single-shot image of the chirped bunch after spectrometer. Higher (lower) energy particles are located on the right (left) side of the image. Projection of this image onto horizontal axis gives CBE spectrum.

As mentioned, energy chirp may not be constant across CBE spectrum (as well as density distribution for the whole bunch is unknown). Besides, CBE spectra for compressed bunch usually contain a very few spikes for DUV-FEL experimental conditions. In this case Fourier analysis of CBE spectra becomes inefficient, due to the fact that bunch form-factor smears out spectral content of modulation, shifted to low-frequency region. Therefore it is more advantageous to perform "local" analysis, taking into account only single modulation period. We assume here, that neither chirp nor longitudinal density do not change along a single modulation period.

In order to perform the "local" analysis of modulation we used the "overmodulated" beam model. The longitudinal phase space of "overmodulated" bunch is sketched on Fig. 1. We choose one of the modulation periods in CBE spectrum and find the distance between two consecutive spikes Δ_E.

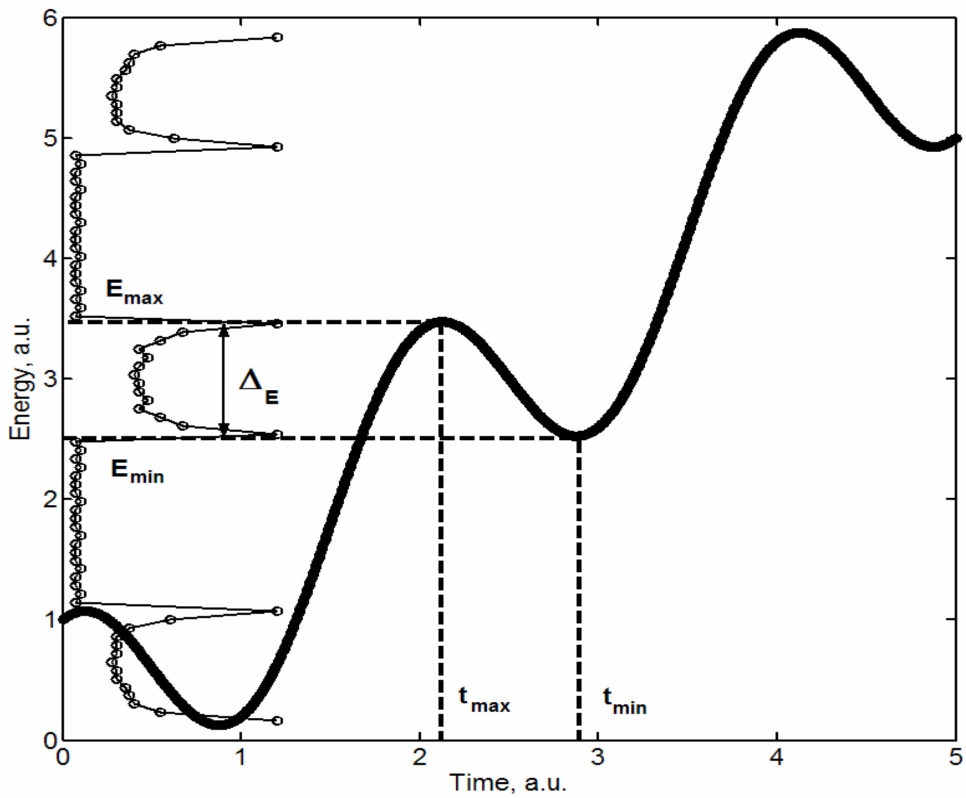

Fig. 3: Longitudinal phase space of "overmodulated" bunch.

Representing energy dependence as

$$E(t) = ht + \Delta\cos(\omega t), \tag{6}$$

we differentiate it in order to obtain minimum and maximum values of energy E_{min} and E_{max}. Corresponding abscissa values are:

$$\begin{aligned} t_{\min} &= \frac{1}{\omega}\arcsin\left(\frac{h}{\Delta\omega}\right), \\ t_{\max} &= \frac{1}{\omega}\left(\pi - \arcsin\left(\frac{h}{\Delta\omega}\right)\right). \end{aligned} \tag{7}$$

And for the energy values we get

$$\begin{aligned} E_{\max} &= a\left(X + \cot(X)\right), \\ E_{\min} &= a\left((\pi - X) - \cot(X)\right), \end{aligned} \tag{8}$$

where $a = \frac{h}{\omega}, X = \arcsin\left(\frac{a}{\Delta}\right)$.

Since $\Delta_E = E_{max} - E_{min}$, we have to solve the following equation, in order to find energy modulation amplitude Δ.

$$X + \cot(X) = \frac{1}{2}\left(\frac{\Delta_E}{a} + \pi\right) \quad (9)$$

Roots of this equation give the energy modulation amplitude. The advantage of this technique is that it does not require knowledge of intrinsic energy spread. Note, that this method works only for "overmodulated" bunch, which constrains the method to be valid only when h/(Δω)>1. For the case h/(Δω)<1 one can use different method [1].

Conclusion

In this paper we discussed correlations in the chirped bunch energy spectrum. Collective effects introduce deviations of energy along the bunch, while it is traveling along accelerator. Since RF gun generates very "cold" electron bunch, intrinsic energy spread does not hide these local energy deviations, mapped into CBE spectrum. Sharp spikes in CBE distribution generate a family of harmonics in the Fourier spectrum. This spectral shape is confirmed by analysis presented here for simplified case of coasting beam with uniform density distribution. In principle, one can retrieve main parameters of the short-wavelength energy modulation, studying the Fourier spectrum. Due to a complicate nonlinear correlation in the bunch longitudinal phase space energy chirp along the bunch is time dependent and usually unknown. That makes direct implementation of Fourier analysis for studying of CBE spectra difficult.

Therefore it is more advantageous to perform a local analysis, relying only on several modulation periods. This can be used for a special case of the "overmodulated" bunch, which is natural for experiment since it does not require large energy chirps. Local method of CBE spectra analysis is used in [9].

Acknowledgements

Authors wishes to thank D. Dowell, A. Doyuran, P.Emma, Z. Huang, S. Krinsky, H. Loos, J.B. Murphy, J. Rose, B. Sheehy, X.J. Wang, J.Wu, Z. Wu for useful discussions and comments. Work is performed under DOE contract DE-AC02-76CH00016.

Bibliography

[1] "Beam energy modulation and its impact on rf zerophasing bunch profile measurement", Z. Huang and T. Shaftan, SLAC-PUB-9788

[2] "Microbunching and beam break-up in DUV-FEL accelerator", T. Shaftan et al., Proc. of PAC 2003 conference, Portland, June 2003, p. 2011

[3] "Bunch compression in the SDL linac", T. Shaftan et al., Proc. of EPAC 2002 Conference, Paris, June 2002

[4] "Ultrashort electron bunch length measurements at DUVFEL", W. Graves et al., PAC 2001, Chicago, p. 2222

[5] “Measurements of femtosecond electron bunches using a rf zero-phasing method”, D.X. Wang et. al., Phys. Rev., E, 57, 2283 (1998)
[6] “Measuring longitudinal distribution and bunch length of femtosecond bunches with RF zero-phasing method”, D. X. Wang and G. A. Krafft, Proc. of PAC 1997 Conference, Vancouver, July 1997, p. 2020
[7] L.H.-Yu, Phys. Rev. A 44 (1991) 5178
[8] “Physics of collective beam instabilities in high energy accelerators”, A. Chao, Wiley-Interscience Publications
[9] “Experimental investigation of a space charge induced modulation in high-brightness electron beam”, T. Shaftan and Z. Huang, to be published