

# SPACE CHARGE SIMULATION METHODS INCORPORATED IN SOME MULTI-PARTICLE TRACKING CODES AND THEIR RESULTS COMPARISON\*

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## Abstract

Space charge in high intensity beams is an important issue in accelerator physics. Due to the complicity of the problems, the most effective way of investigating its effect is by computer simulations. In the recent years, many space charge simulation methods have been developed and incorporated in various 2D or 3D multi-particle-tracking codes. It has become necessary to benchmark these methods against each other, and against experimental results. As a part of global effort, we present our initial comparison of the space charge methods incorporated in simulation codes ORBIT++ [1], ORBIT [2-5] and SIMPSONS [6,7]. In this paper, the methods included in these codes are overviewed. The simulation results are presented and compared. Finally, from this study, the advantages and disadvantages of each method are discussed.

## 1 INTRODUCTION

Space charge force is one of the major sources of emittance growth, halo/tail formation and beam loss. Because of its nature of electromagnetic force proportional to  $1/\gamma^2$ , it is significant only at low energy. Space charge force is a function of beam size in transverse and longitudinal directions as well as number of particles in a bunch. Once the beam emittance growth occurs, the beam is no longer affected by the same space charge force as the one before. In the case of multi-turn injection, the number of particles is gradually increased as a function of turn number throughout the process. In addition, the transverse emittance is sometimes also manipulated by the so-called phase space painting. Moreover, the peak intensity and the length of a bunch strongly depend on the RF voltage pattern applied. All of these make the analysis of space charge effects difficult. Therefore, it seems that computer simulation is the only way to model the effects and analyze beam behavior in a self-consistent way.

In the last two decades, many computer codes for multi-particle multi-dimensional beam simulation have been developed with different degree of completeness. They include ACCSIM [8], TRACK3D [9], WARP3D [10], SIMPSONS [6,7] and ORBIT [2-5]. A complete simulation code can be characterized by full 6-D phase

space tracking of a beam propagating through a machine lattice, in the presence of particle-to-particle and particle-to-wall interactions. Normally, the machine lattice is represented by matrices and higher order transformations obtained by an external optics code (such as MAD, DIMAD, SYNCH and TRANSPORT). The particle-to-particle and particle-to-wall interactions are calculated with PIC (Particle In Cell) algorithms. The space charge forces or potential are obtained by solving Poisson equation with different strategies. Each strategy may have its advantage and disadvantage. They may also present slightly different results.

In the recent years, the activities in higher intensity accelerator designs have made stronger demand in the space charge code validations. The simulation codes should be compared to experimental observations and available theories, as well as benchmarked against each other. It has been initiated to establish the collaboration on this subject among all interested parties in various laboratories worldwide. As a part of global effort, we present our initial comparison of the space charge methods incorporated in simulation codes ORBIT++ [1], ORBIT [2-5] and SIMPSONS [6,7]. In this paper, the methods included in this study are overviewed. The simulation results are presented and compared. Finally, from this study, the advantages and disadvantages of each method are discussed.

## 2 POISSON EQUATION SOLVERS USED BY ORBIT++

Space Charge forces are calculated in ORBIT by solving the Poisson equation, in either differential form

$$\nabla^2\Phi = -\frac{\rho}{\epsilon_0\gamma^2}, \quad (1)$$

or in integral form

$$\Phi(P) = \frac{1}{4\pi\epsilon_0\gamma^2} \int \frac{\rho(Q)}{r} dQ, \quad (2)$$

where  $\Phi$  the electric potential,  $P$  a field point and  $Q$  a source point. The factor  $\gamma^{-2}$ , with  $\gamma$  the particle total energy in terms of the mass energy, represents the partial compensation of the electric repulsion between charges and the magnetic attraction between current elements, in the assumption (generally well satisfied in a synchrotron) that the beam bunch is much longer than wide. The

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elliptic differential Equation is solved by difference methods, by writing the Laplacian on a finite  $M \times N$  mesh extending to the walls, and solving the resulting linear system with boundary conditions by at least two methods:

- (1) *LU decomposition* of the  $M^2 \times N^2$  matrix, matrix compaction and matrix multiplication; and
- (2) *SOR (Successive Over Relaxation) Method* [11] for iteration. Iteration is natural, since the beam transverse charge density profile changes slowly from one space charge node to the next.

The integral equation (2) is also solved with two competitive methods:

- (1) *Brute Force Method* is a direct integration. It is lengthy due to its physically very transparent algorithm; and
- (2) *Convolution (FFT) Method* [12] reduces the integral to a convolution. First, the charge density  $\rho$  and the Green function are Fourier transformed from the space domain to the frequency domain, then the two transforms are multiplied and finally the potential is obtain by inverse transform.

### 3 POISSON EQUATION SOLVER USED BY SIMPSONS

In SIMPSONS, the Poisson equation in differential form (1) is solved with a boundary condition of perfectly conducting circular pipe in cylindrical coordinates for a 3-D beam or in polar coordinate for 2-D beam:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \varphi^2} = -\frac{\rho}{\epsilon_0 \gamma^2} \quad (3)$$

Once a charge distribution at grid points is obtained, it is Fourier transformed in the azimuthal direction:

$$\Phi = \sum_m \phi_m \exp(im\varphi), \quad (4)$$

$$\frac{\rho}{\epsilon_0 \gamma^2} = \sum_m n_m \exp(im\varphi). \quad (5)$$

with the inverse transform,

$$\phi_m = \frac{1}{2\pi} \int_0^{2\pi} \Phi(r, z, \varphi) \exp(-im\varphi) d\varphi, \quad (6)$$

$$n_m = \frac{1}{2\pi} \int_0^{2\pi} \frac{\rho(r, z, \varphi)}{\epsilon_0 \gamma^2} \exp(-im\varphi) d\varphi. \quad (7)$$

Note that,  $n_{-m} = n_m^*$ , then,

$$n_m(r, z) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \phi_m(r, z) \right) - \frac{m^2}{r^2} \phi_m(r, z). \quad (8)$$

The general solution to the equation ( $m \geq 0$ ) is:

$$\phi_m = \alpha \left( \frac{r}{b} \right)^m + W(r) - \left( \frac{r}{b} \right)^m W(b) \quad (9)$$

With the boundary condition  $\phi_m = 0$  at  $r = b$  ( $b$  is the beam pipe radius), it becomes

$$\phi_m = W(r) - \left( \frac{r}{b} \right)^m W(b), \quad (10)$$

where

$$W(r, z) = \int \ln \frac{r}{r'} n_m(r', z) r' dr' \quad \text{for } m=0 \quad (11)$$

$$W(r, z) = \frac{r^m}{2m} \int n_m(r', z) r'^{(1-m)} dr' - \frac{r^{-m}}{2m} \int n_m(r', z) r'^{(1+m)} dr' \quad \text{for } m \neq 0 \quad (12)$$

where  $\phi_{-m} = \phi_m^*$  is used to compute  $\phi_m$  for  $m < 0$ . In practice, the integral is replaced with summation at grid points. The electric field differentiations are done analytically beforehand (except with  $z$ ):

$$E_r = \sum_m \frac{\partial \phi_m}{\partial r} \exp(im\varphi) \quad (13)$$

$$E_\varphi = \sum_m \frac{1}{r} \frac{\partial \phi_m}{\partial \varphi} \exp(im\varphi) \quad (14)$$

$$E_z = \sum_m \frac{\partial \phi_m}{\partial z} \exp(im\varphi) \quad (15)$$

### 4 COMPARISON OF RESULTS

As described in Section 2, many Poisson equation solvers are included in ORBIT++. However, the two most powerful methods in ORBIT++ are SOR method and convolution (FFT) method. In this work, the two methods are compared under the identical physical conditions of the Rapid Cycling Medical Synchrotron (RCMS) [13]. All the physical quantities used in the simulations are chosen to be as close as possible to the specifications in the current design. The optical lattice and the salient parameters used in the simulations can be found from references [14,15]. The maximum and RMS emittance comparison are presented by Fig. 1 and Fig. 2. The comparisons of FFT method and the SIMPSONS method are made with the identical physical and numerical conditions [16,17] of original FODO lattice of SNS accumulator ring. Figs. 3-5 show comparisons of beam emittance distributions obtained from ORBIT FFT method and SIMPSONS method in a full-intensity beam with KV, Waterbag and Gaussian distribution respectively.

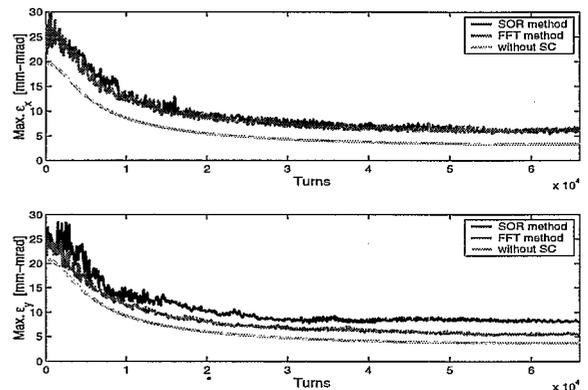


Fig. 1 Maximum horizontal (top figure) and vertical (bottom figure) in RCMS ring obtained from SOR method and FFT Method compared to without space charge.

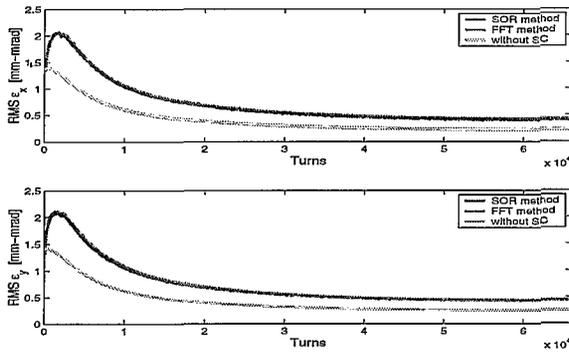


Fig. 2 RMS horizontal (top figure) and vertical (bottom figure) in RCMS ring obtained from SOR and FFT Method in ORBIT++ compared to without space charge.

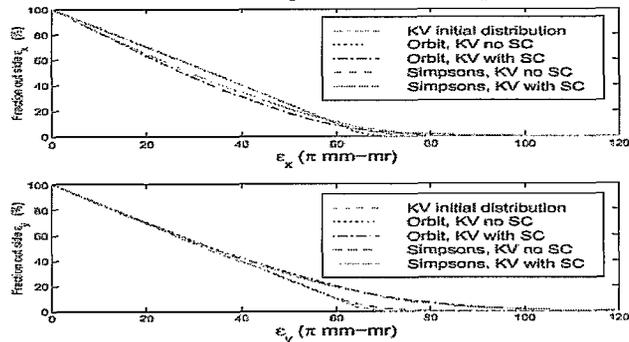


Fig. 3 Comparison of particle emittance distributions in a full-intensity beam with a KV distribution obtained from ORBIT (FFT method) and SIMPSONS.

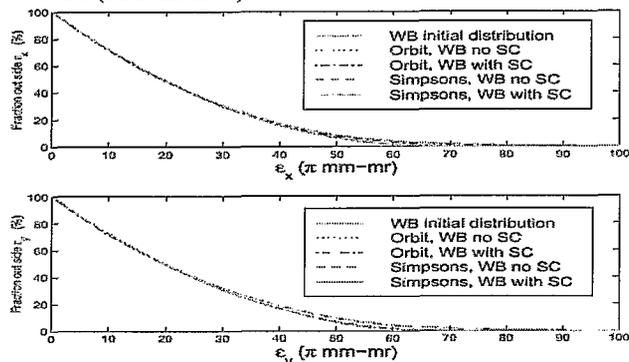


Fig. 4 Comparison of particle emittance distributions in a full-intensity beam with a Waterbag distribution obtained from ORBIT (FFT method) and SIMPSONS.

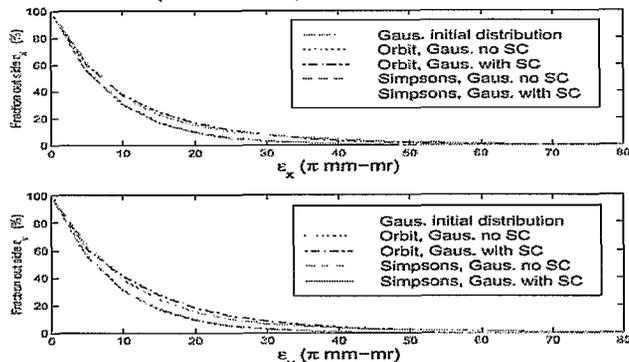


Fig. 5 Comparison of particle emittance distributions in a full-intensity beam with a Gaussian distribution obtained from ORBIT (FFT method) and SIMPSONS.

## 5 CONCLUSION AND DISCUSSION

The results of this study have demonstrated good agreements between the SOR and convolution method in ORBIT++, as well as the method in SIMPSONS. Very good understanding of these complex codes and a minimum  $64 \times 64$  transverse grid points are necessities. A smoothing parameter is introduced in the denominator to avoid singularities that arise when an individual representative macro-particle is too close to another particle in the integral formalism. Systematic simulations have shown that the importance of a smoothing factor decreases while the number of macro-particles increases. The convolution method takes less CPU time. However, it could not include a conductive wall, as in most of realistic systems. While the SOR method allows to specify boundary conditions of any shape. SIMPSONS assumes a boundary condition of a perfectly conducting circular pipe, which often is a good approximation of beam lines. Also, the nature of Poissons equation solver in SIMPSONS makes it convenient for studies by modes.

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