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EVENT BY EVENT AVERAGES IN HEAVY ION COLLISIONS

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NA49 (Pb+Pb, CERN), PHENIX and STAR (Au+Au, BNL) have presented measurements of the event-by-event average p_T (denoted M_{p_T}) in relativistic heavy ion collisions. Event-by-event averages are most useful to resolve the case of two or several classes of events with e.g. different temperature parameters. The distribution of M_{p_T} is discussed, with emphasis on the case of statistically independent emission according to the semi-inclusive p_T and charged multiplicity distributions. Deviations from statistically independent emission are quantified in terms of a simple two component model, with the individual components being Gamma distributions.

1 Semi-Inclusive Distributions

In p-p and heavy ion collisions, the semi-inclusive distributions of charged multiplicity, n, and transverse momentum, p_T, for a given centrality class (or impact parameter), summing over all events in a class, are typically Negative Binomial Distributions (NBD) and Gamma Distributions, respectively.[1,2] The NBD is the first departure from a Poisson distribution (for repeated independent trials, each with the same probability for a given outcome) which forms the basis of most physicist's 'intuition' for the statistics of random processes. The NBD allows some correlation, which is represented by a parameter parameter, $1/k$, which is zero for a Poisson distribution:

$$\frac{1}{k} = \frac{\sigma_n^2}{\langle n \rangle^2} - \frac{1}{\langle n \rangle} \qquad , \tag{1}$$

where $\langle x \rangle$ is the mean and σ_x^2 is the variance (σ_x is the standard deviation) of the quantity x:

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 \qquad . \tag{2}$$

The Gamma distribution has particularly simple properties under convolutions and scale transformations and for these reasons has proved useful in the study of 'E_T' distributions.[3] The

Gamma distribution is a function of a continuous variable x and has paramters p and b

$$f(x) = f_\Gamma(x,p,b) = \frac{b}{\Gamma(p)}(bx)^{p-1}e^{-bx} \tag{3}$$

where $p > 0$, $b > 0$, $0 \leq x \leq \infty$, $\Gamma(p) = (p-1)!$ if p is an integer, and $f(x)$ is normalized, $\int_0^\infty f(x)dx = 1$. The n-fold convolution of $f_\Gamma(x,p,b)$ is simply $f_\Gamma(x,np,b)$.

The semi-inclusive single particle p_T distribution is typically a Gamma distribution with $p = 2$, and the parameters of the distribution can be derived from the semi-inclusive mean and standard deviation, $\langle p_T \rangle$ and σ_{p_T} :

$$p = \frac{\langle p_T \rangle^2}{\sigma_{p_T}^2} \qquad b = \frac{\langle p_T \rangle}{\sigma_{p_T}^2} \quad . \tag{4}$$

2 Event-by-Event averages

In heavy ion collisions, events in a given semi-inclusive class may contain many particles so it has become popular [4] to study the distribution of the average of a quantity x over the n particles for each event, the event-by-event average:

$$\bar{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad . \tag{5}$$

For $x = p_T$, the event-by-event average transverse momentum has been denoted [4] M_{p_T} where n also varies from event-to-event.

3 "It's not a Gaussian it's a Gamma distribution"

For statistically independent emission an analytical formula for the distribution in M_{p_T} can be derived using the convolution property of the Gamma distribution for the sum of statistically indendent samples from a given population. [2] It depends on the 4 semi-inclusive parameters $\langle n \rangle$, $1/k$, b and p which are derived from the quoted means and standard deviations of the semi-inclusive p_T and multiplicity distributions (Eqs. 1, 4)

$$f(y) = \sum_{n=n_{\rm min}}^{n_{\rm max}} f_{\rm NBD}(n, 1/k, \langle n \rangle)\, f_\Gamma(y, np, nb) \qquad\qquad \text{where} \;\; y = M_{p_T}. \tag{6}$$

The result is in excellent agreement with the NA49 Pb+Pb-central measurement [4] and also with the new PHENIX central measurement in Au+Au at $\sqrt{s_{NN}} = 130$ GeV at RHIC. [5] (see Fig. 1). Also the Gamma distribution shape is now obvious for the less central data.

3.1 Mixed Events as the Random Baseline

In Fig. 1-*right*, 'Mixed Events' are used as the random baseline reference, since Eq. 6 is only an (excellent) approximation and the deviation of the data from the random baseline is very very small. It is important to note that the Mixed Events must use exactly the same n distribution as the data and match the inclusive $\langle p_T \rangle$ to high precision [5] since $\langle M_{p_T} \rangle = \langle p_T \rangle \equiv \mu$.

4 How to quantify the non-random effect?

4.1 Moments—So far, only Variance or Standard Deviation

The very small if any non-random effect in Fig. 1 can be quantified simply in terms of the difference between the variances or standard deviations of the data and the random baseline:

$$\left(\frac{\sigma_{\bar{x}}^2}{\mu^2} - \frac{1}{n}\frac{\sigma_x^2}{\mu^2}\right) / \frac{1}{n}\frac{\sigma_x^2}{\mu^2} \quad \text{or} \quad \left(\frac{\sigma_{\bar{x}}}{\mu} - \frac{1}{\sqrt{n}}\frac{\sigma_x}{\mu}\right) / \frac{1}{\sqrt{n}}\frac{\sigma_x}{\mu} = F \qquad , \tag{7}$$

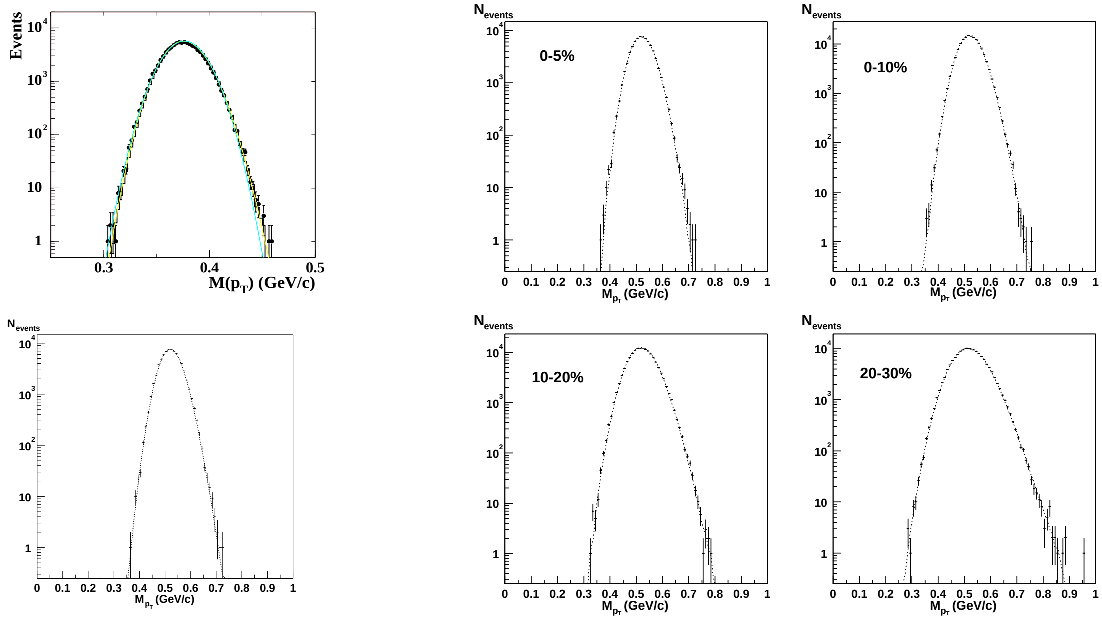

Figure 1: *top left:* Gamma Distribution for M_{p_T} (light line) compared to Gaussian with same $\langle p_T \rangle$ and $\sigma_{M_{p_T}}$ (darker line) for NA49 measurement. *bottom left:* Eq. 6 with PHENIX Au+Au central (top 5%) data. *right:* PHENIX Au+Au data for $\Delta\Phi = 1.02$, $|\eta| < 0.35$ vs centrality. The dotted curves are mixed event distributions.

where for the random case $\sigma^2_{\bar{x}} = \sigma^2_x/n$. Groups argue over whether the variance or standard deviation is better, however for small effects, these measures are equivalent:

$$\frac{\Delta\sigma^2}{\sigma^2} = 2\,\frac{\Delta\sigma}{\sigma} = 2F \qquad . \tag{8}$$

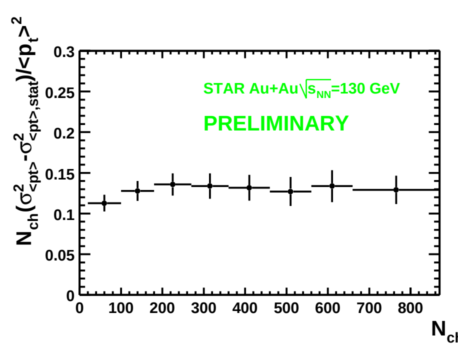

Fig. 2: Star results for F vs n, assuming $\sigma^2_{\langle p_T\rangle,\mathrm{stat}}/\mu^2 = 1/(np) = 1/2n$, in which case the label on the y axis is equal to F

In terms of F, the fractional difference of the data and random standard deviations, the PHENIX results from Fig. 1 for centralities 0–5% ($\langle n\rangle = 59.6$), 0–10% (53.9), 10–20% (36.6) and 20–30% (25.0) are $F = 0.019 \pm 0.021$, 0.020 ± 0.025, 0.021 ± 0.022, 0.018 ± 0.030 respectively, very small indeed, where the error is dominantly systematic. Note that F is independent of centrality, an effect also observed by STAR [6] (Fig. 2), although for the same value of $\langle n\rangle$ the STAR preliminary result for F is 6 times larger than PHENIX, possibly due to the larger solid angle of the measurement in STAR ($\Delta\Phi = 2\pi$, $|\eta| < 0.75$). A starkly different dependence of F on $\langle n\rangle$ is predicted for the case [7] of a temperature parameter $T = 1/b$ which varies event-by-event with mean and variance, $\langle T\rangle$, σ^2_T, clearly not observed by either PHENIX or STAR:

$$F = \frac{p}{2}(\langle n\rangle - 1)\frac{\sigma^2_T}{\langle T\rangle^2} \qquad . \tag{9}$$

4.2 Two-Component Model for the M_{p_T} distribution

The event-by-event average is most useful to resolve the case of e.g. 2 classes of events with different p_T distributions of which only one component appears on any given event. We represent

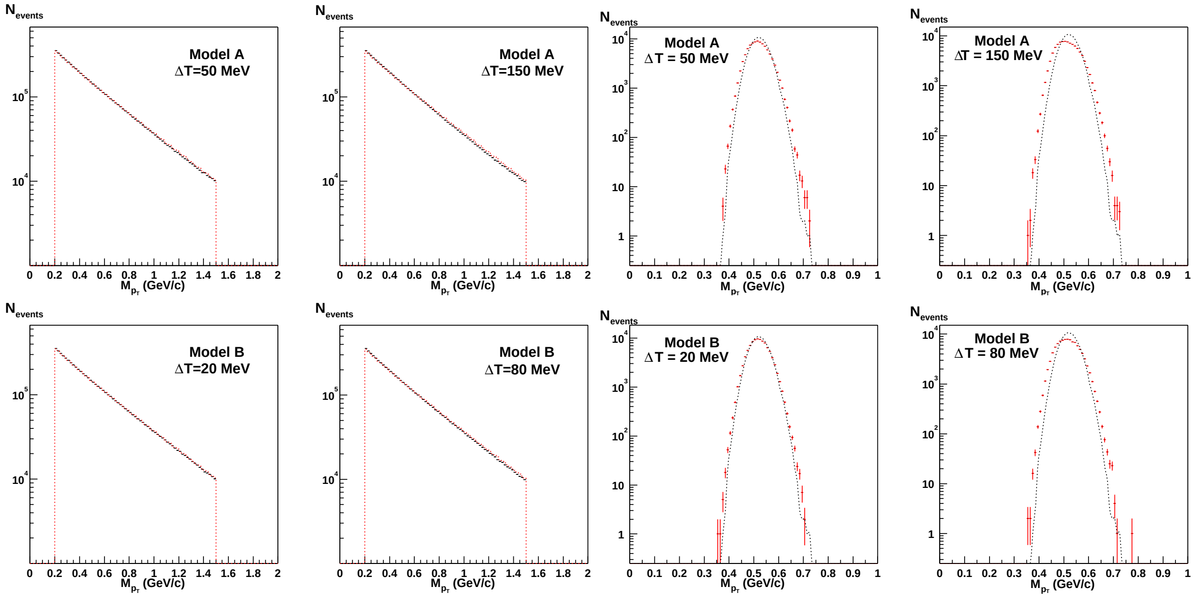

Figure 3: *left:* Semi-inclusive p_T distribution for data and models. *right:* Comparison of the PHENIX mixed event distribution for 0–5% centrality from Fig. 1 (dotted) to the 2-component models (points).

the component semi-inclusive distributions as Gamma distributions:

$$f_c(p_T) = q f_\Gamma(p_T, p_1, b_1) + (1-q) f_\Gamma(p_T, p_2, b_2), \tag{10}$$

where q and $1-q$ are the probabilities for an event to have either component distribution. We constrain the compound distribution to be as close to the observed semi-inclusive distribution by constraining its mean μ_c and variance $\sigma^2_{x_c}$ to be equal to the observed semi-inclusive values μ, σ^2_x, i.e. $\langle p_T \rangle = \mu_c = q\mu_1 + (1-q)\mu_2 = \mu$ and

$$\frac{\sigma^2_{x_c}}{\mu^2} - \frac{\sigma^2_x}{\mu^2} = 0 = q[\frac{\mu_1^2}{\mu^2}(1+\frac{1}{p_1})] + (1-q)[\frac{\mu_2^2}{\mu^2}(1+\frac{1}{p_2})] - (1+\frac{1}{p}) \quad . \tag{11}$$

We consider two models with $\Delta T \equiv 1/b_2 - 1/b_1$: Model A, where the two components have the same $\mu_{1,2}$ and different $\sigma_{1,2}$; Model B, with different $\mu_{1,2}$ same $\sigma_{1,2}$. Eqs. 10,11 are sufficient to produce compound semi-inclusive distributions which are indistinguishable from the observed distribution (see Fig. 3) yet give M_{p_T} distributions which are obviously no longer simple Gamma distributions. 95%-confidence limits for ΔT as a function of q are obtained by a likelihood-ratio test with result: $\Delta T < 40$ MeV (Model A), $\Delta T < 20$ MeV (Model B), for $q \sim 0.2 - 0.8$.

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References

1. e.g. see reference 2 and references therein.
2. M. J. Tannenbaum, *Phys. Lett.* B **498**, 29 (2001).
3. M. J. Tannenbaum, *Int. J. Mod. Phys.* A **4**, 3377 (1989).
4. H. Appelshäuser, *et al.*, NA49 Collab., *Phys. Lett.* B **459**, 679 (1999).
5. K. Adcox, *et al.*, PHENIX collab, submitted to *Phys. Rev.* C, `nucl-ex/0203015`.
6. S. A. Voloshin, *et al.*, STAR collab, `nucl-ex/0109006`.
7. R. Korus, *et al.*, *Phys. Rev.* C **64**, 054908 (2001).