

Rare K decays: results and prospects

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Abstract. Recent results on rare kaon decays are reviewed and prospects for future experiments are discussed.

INTRODUCTION

In recent years the study of the rare decays of kaons has had three primary motivations. The first is the search for physics beyond the Standard Model (BSM). Virtually all attempts to redress the theoretical shortcomings of the Standard Model (SM) predict some degree of lepton flavor violation (LFV). Decays such as $K_L \rightarrow \mu^\pm e^\mp$ have very good experimental signatures and can consequently be pursued to remarkable sensitivities. These sensitivities correspond to extremely high energy scales in models where the only suppression is that of the mass of the exchanged field. There are also theories that predict new particles created in kaon decay or the violation of symmetries other than lepton flavor.

The second is the potential of decays that are allowed but that are extremely suppressed in the SM. In several of these, the leading component is a G.I.M.-suppressed[1] one-loop process that is quite sensitive to fundamental SM parameters such as V_{td} . These decays are also potentially very sensitive to BSM physics.

Finally there are a number of long-distance-dominated decays which can test theoretical techniques such as chiral Lagrangians that purport to explain the low-energy behavior of QCD. Knowledge of some of these decays is also needed to extract more fundamental information from certain of the one-loop processes.

This field is quite active as indicated by Table 1 that lists the decays for which results have been forthcoming in the last couple of years as well as those that are under analysis. Thus in a short review such as this, one must be quite selective.

BEYOND THE STANDARD MODEL

There were several K decay experiments dedicated to lepton flavor violation at the Brookhaven AGS during the 1990's. These advanced the sensitivity to such processes by many orders of magnitude. In addition, several "by-product" results on LFV and other BSM topics have emerged from the other kaon decay experiments of this period. Table 2 summarizes the status of work on BSM probes in kaon decay.

TABLE 1. Rare K decay modes under recent or on-going study.

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$K_L \rightarrow \pi^0 \nu \bar{\nu}$	$K_L \rightarrow \pi^0 \mu^+ \mu^-$	$K_L \rightarrow \pi^0 e^+ e^-$
$K^+ \rightarrow \pi^+ \mu^+ \mu^-$	$K^+ \rightarrow \pi^+ e^+ e^-$	$K_L \rightarrow \mu^+ \mu^-$	$K_L \rightarrow e^+ e^-$
$K^+ \rightarrow \pi^+ e^+ e^- \gamma$	$K^+ \rightarrow \pi^+ \pi^0 \nu \bar{\nu}$	$K_L \rightarrow e^\pm e^\mp \mu^\pm \mu^\mp$	$K^+ \rightarrow \pi^+ \pi^0 \gamma$
$K_L \rightarrow \pi^+ \pi^- \gamma$	$K_L \rightarrow \pi^+ \pi^- e^+ e^-$	$K^+ \rightarrow \pi^+ \pi^0 e^+ e^-$	$K^+ \rightarrow \pi^0 \mu^+ \nu \gamma$
$K_L \rightarrow \pi^0 \gamma \gamma$	$K^+ \rightarrow \pi^+ \gamma \gamma$	$K^+ \rightarrow \mu^+ \nu \gamma$	$K^+ \rightarrow e^+ \nu e^+ e^-$
$K^+ \rightarrow \mu^+ \nu e^+ e^-$	$K^+ \rightarrow e^+ \nu \mu^+ \mu^-$	$K_L \rightarrow e^+ e^- \gamma$	$K_L \rightarrow \mu^+ \mu^- \gamma$
$K_L \rightarrow e^+ e^- \gamma \gamma$	$K_L \rightarrow \mu^+ \mu^- \gamma \gamma$	$K_L \rightarrow e^+ e^- e^+ e^-$	$K_L \rightarrow \pi^0 e^+ e^- \gamma$
$K^+ \rightarrow \pi^+ \mu^+ e^-$	$K_L \rightarrow \pi^0 \mu^\pm e^\mp$	$K_L \rightarrow \mu^\pm e^\mp$	$K^+ \rightarrow \pi^- \mu^+ e^+$
$K^+ \rightarrow \pi^- e^+ e^+$	$K^+ \rightarrow \pi^- \mu^+ \mu^+$	$K^+ \rightarrow \pi^+ X^0$	$K_L \rightarrow e^\pm e^\pm \mu^\mp \mu^\mp$
$K^+ \rightarrow \pi^+ \gamma$			

TABLE 2. Current 90% CL limits on K decay modes violating the SM. The violation codes are “LF” for lepton flavor, “LN” for lepton number, “G” for generation number, [9], “H” for helicity, “N” requires new particle

Process	Violates	Limit	Experiment	Reference
$K_L \rightarrow \mu e$	LF	4.7×10^{-12}	AGS-871	[2]
$K^+ \rightarrow \pi^+ \mu^+ e^-$	LF	2.8×10^{-11}	AGS-865	[3]
$K^+ \rightarrow \pi^+ \mu^- e^+$	LF, G	5.2×10^{-10}	AGS-865	[4]
$K_L \rightarrow \pi^0 \mu e$	LF	4.4×10^{-10}	KTeV	[5]
$K^+ \rightarrow \pi^- e^+ e^+$	LN, G	6.4×10^{-10}	AGS-865	[4]
$K^+ \rightarrow \pi^- \mu^+ \mu^+$	LN, G	3.0×10^{-9}	AGS-865	[4]
$K^+ \rightarrow \pi^- \mu^+ e^+$	LF, LN, G	5.0×10^{-10}	AGS-865	[4]
$K_L \rightarrow \mu^\pm \mu^\pm e^\mp e^\mp$	LF, LN, G	1.36×10^{-10}	KTeV	[6]
$K^+ \rightarrow \pi^+ f^0$	N	1.1×10^{-10}	AGS-787	[7]
$K^+ \rightarrow \pi^+ \gamma$	H	3.6×10^{-7}	AGS-787	[8]

It is clear from this table that any deviation from the SM must be highly suppressed. The LFV probes in particular have become the victims of their own success. The specific theories they were designed to test have been killed or at least forced to retreat to the point where meaningful tests in the kaon system would be very difficult. Both kaon flux and rejection of background are becoming problematical. Analysis of data already collected is continuing but no new kaon experiments focussed on LFV are being planned. Interest in probing LFV has migrated to the muon sector.

ONE LOOP DECAYS

In the kaon sector experimental effort has shifted from LFV to “one-loop” decays. These are GIM-suppressed decays in which loops containing weak bosons and heavy quarks dominate or at least contribute measurably to the rate. These processes include $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \mu^+ \mu^-$, $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$. In some cases the one-loop contributions violate CP. In one, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, this contribution completely dominates the decay[10]. Since the GIM-mechanism tends to enhance the contribution of heavy quarks in the loops, in the SM these decays are sensitive to the product of

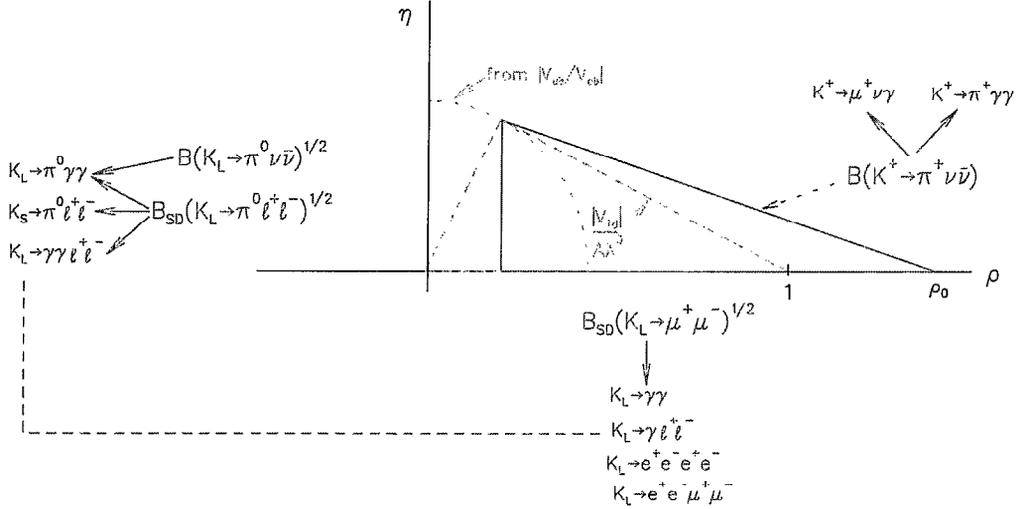


FIGURE 1. K decays and the unitarity plane. The usual unitarity triangle is dashed. The triangle that can be constructed from rare K decays is solid. See text for further details.

couplings $V_{ts}^* V_{td}$, often abbreviated as λ_t . Although as discussed below one can readily analyze these decays in terms of the real and imaginary parts of λ_t , for comparison with results in the B system, it is convenient to parameterize them in terms of the Wolfenstein variables, A , ρ , and η . Fig. 1 shows the relationship of rare kaon decays to the unitarity triangle construction. The dashed triangle is the usual one whereas the solid triangle illustrates the information available from rare kaon decays. Note that the “unitarity point”, (ρ, η) can be determined from either triangle, and any disagreement between the K and B determinations indicates physics beyond the SM. In Fig. 1 the branching ratio closest to each side of the triangle determines the length of that side. The arrows leading outward from those branching ratios point to processes that need to be studied either because they potentially constitute backgrounds, or because knowledge of them is required to relate the innermost branching ratios to fundamental parameters. $K_L \rightarrow \mu^+ \mu^-$, which can determine the bottom of the triangle (ρ), is the process for which the experimental data is the best, but for which the theory is most problematical. $K_L \rightarrow \pi^0 \nu \bar{\nu}$, which determines the height of the triangle is theoretically the cleanest, but experiment is many orders of magnitude short of the SM-predicted level. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, which determines the hypotenuse, is nearly as clean and has been observed. Prospects for this latter process are probably the best of the three since it is already clear it can be exploited.

$$K_L \rightarrow \mu^+ \mu^-$$

The short distance component of this decay can be quite reliably calculated in the SM[11]. The most recent measurement of its branching ratio[12] based on some 6200 events gave $B(K_L \rightarrow \mu^+ \mu^-) = (7.18 \pm 0.17) \times 10^{-9}$. However $K_L \rightarrow \mu^+ \mu^-$ is dominated by long distance effects, the largest of which, the absorptive contribution mediated by

$$\begin{aligned}
\frac{B_{\gamma\gamma}^{abs}(K_L \rightarrow \mu\mu)}{B(K_L \rightarrow \pi^+\pi^-)} &= \frac{\text{calculated}}{\downarrow} \frac{B_{\gamma\gamma}^{abs}(K_L \rightarrow \mu\mu)}{B(K_L \rightarrow \gamma\gamma)} \frac{\text{last measured by N 431 in 1987}}{\downarrow} \frac{B(K_L \rightarrow \gamma\gamma)}{B(K_L \rightarrow \pi^0\pi^0)} \frac{\text{from several exp. last in 1976!}}{\downarrow} \frac{B(K_S \rightarrow \pi^0\pi^0)}{B(K_S \rightarrow \pi^+\pi^-)} \left(1 - 6\text{Re}\frac{\epsilon'}{\epsilon}\right) \\
&= \frac{1.2 \cdot 10^{-5}}{\downarrow} \frac{0.632 \pm 0.009}{\downarrow} \frac{(2.185 \pm 0.027)^{-1}}{\downarrow} \frac{1 - 6(17.2 \pm 1.8) \cdot 10^{-4}}{\downarrow}
\end{aligned}$$

FIGURE 2. Components of the calculation of $\frac{B_{\gamma\gamma}^{abs}(K_L \rightarrow \mu^+\mu^-)}{B(K_L \rightarrow \pi^+\pi^-)}$.

$K_L \rightarrow \gamma\gamma$, accounts for $(7.07 \pm 0.18) \times 10^{-9}$. Subtracting the two, yields a 90% CL upper limit on the total dispersive part of $B(K_L \rightarrow \mu^+\mu^-)$ of 0.37×10^{-9} . One can do a little better than this in the following way. The actual quantity measured in Ref [12] was $\frac{B(K_L \rightarrow \mu^+\mu^-)}{B(K_L \rightarrow \pi^+\pi^-)} = (3.48 \pm 0.05) \times 10^{-6}$. One wants to subtract from this the quantity $\frac{B_{\gamma\gamma}^{abs}(K_L \rightarrow \mu^+\mu^-)}{B(K_L \rightarrow \pi^+\pi^-)}$. Fig 2 shows the components of this latter ratio, obtained from Ref. [13], whose product is $(3.435 \pm 0.065) \times 10^{-6}$.

Subtracting this from $\frac{B(K_L \rightarrow \mu^+\mu^-)}{B(K_L \rightarrow \pi^+\pi^-)}$ yields $\frac{B^{disp}(K_L \rightarrow \mu^+\mu^-)}{B(K_L \rightarrow \pi^+\pi^-)} = (0.045 \pm 0.082) \times 10^{-6}$ (where B^{disp} refers to the dispersive part of $B(K_L \rightarrow \mu^+\mu^-)$). This can then be multiplied by $B(K_L \rightarrow \pi^+\pi^-) = (2.056 \pm 0.033) \times 10^{-3}$ to obtain $B^{disp}(K_L \rightarrow \mu^+\mu^-) = (0.093 \pm 0.169) \times 10^{-9}$, or $B^{disp}(K_L \rightarrow \mu^+\mu^-) < 0.31 \times 10^{-9}$ at 90% CL. Note that some of the components represent quite old measurements. Since $B(K_L \rightarrow \mu^+\mu^-)$ and $B_{\gamma\gamma}^{abs}(K_L \rightarrow \mu^+\mu^-)$ are so close, small shifts in the component values could have relatively large consequences for $B^{disp}(K_L \rightarrow \mu^+\mu^-)$. Several of the components could be remeasured by experiments presently in progress ¹. Now if one inserts the result of even very conservative recent CKM fits into the formula for the short distance part of $B(K_L \rightarrow \mu^+\mu^-)$, one gets rather poor agreement with the limit of $B^{disp}(K_L \rightarrow \mu^+\mu^-)$ derived above. For example the 95% CL fit of Hocker et al.[15][16], $\bar{\rho} = 0.04 - 0.37$, gives $B^{SD}(K_L \rightarrow \mu^+\mu^-) = (0.4 - 2.5) \times 10^{-9}$. So why haven't we been hearing about this apparent violation of the SM?

The answer is that unfortunately $K_L \rightarrow \gamma^*\gamma^*$ also gives rise to a dispersive contribution, which is much less tractable than the absorptive part, and which can interfere with the short-distance weak contribution that one is trying to extract. The problem in calculating this contribution is the necessity of including intermediate states with virtual photons of

¹ There is a new preliminary result from the KLOE experiment of $B(K_S \rightarrow \pi^+\pi^-)/B(K_S \rightarrow \pi^0\pi^0) = 2.192 \pm 0.003_{stat} \pm 0.016_{syst}$ [14]. Fortunately, inserting this result in place of the PDG value makes no difference to the final limit.

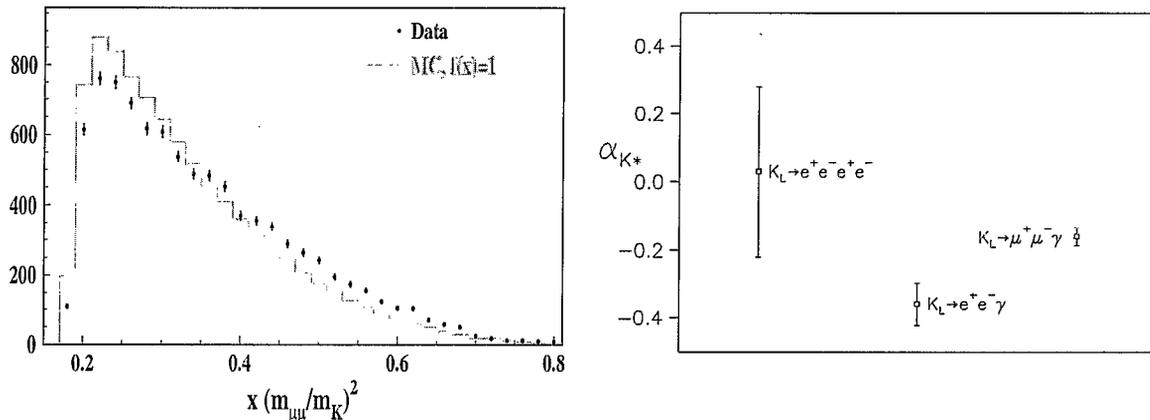


FIGURE 3. Left: Spectrum of $x = (m_{\mu\mu}/m_K)^2$ in $K_L \rightarrow \mu^+\mu^-\gamma$ from Ref.[17]. Right: Determinations of the BMS parameter α_{K^*} from three K_L decays involving virtual photons.

all effective masses. Thus such calculations can only partially be validated by studies of processes containing virtual photons in the final state. Recently there have been publications on $K_L \rightarrow \gamma\mu^+\mu^-$ [17] (9327 events), $K_L \rightarrow e^+e^-e^+e^-$ [18] (441 events), and $K_L \rightarrow \mu^+\mu^-e^+e^-$ [6] (38 events) and there exist slightly older high statistics data on $K_L \rightarrow \gamma e^+e^-$ [19] (6864 events). Figure 3-left shows the spectrum of $x = (m_{\mu\mu}/m_K)^2$ from Ref.[17]. The disagreement between the data (filled circles with error bars) and the prediction of pointlike behavior (histogram) clearly indicates the presence of a form factor. A long-standing candidate for this is provided by the BMS model[20] which depends on a single parameter, α_{K^*} .

Fig. 3-right shows three determinations of this parameter. The level of agreement of these results leaves something to be desired. Fitting to a more recent parameterization of these decays[21] also results in a very marginal degree of agreement. This may improve when radiative corrections are properly taken into account. Thus additional effort, both experimental and theoretical, is required before the very good data on $B(K_L \rightarrow \mu^+\mu^-)$ can be fully exploited.

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ suffers from none of the long distance complications of $K_L \rightarrow \mu^+\mu^-$. Theoretically it is remarkably clean. The hadronic matrix element, so often a problem in other processes, can be calculated to a $\sim 2\%$ via an isospin transformation from that of K_{e3} [22]. Interest in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is driven in large part by its sensitivity to V_{td} (it is actually directly sensitive to the quantity $|V_{ts}^* V_{td}|$). Its amplitude is proportional to the dark slanted line at the right in Fig. 1. This is equal to the vector sum of the line proportional to $|V_{td}|/A\lambda^3$ (where $\lambda \equiv \sin\theta_{Cabibbo}$) and that from (1,0) to the point marked ρ_0 . The length $\rho_0 - 1$ along the real axis is proportional to the amplitude for the charm contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. The QCD corrections to this amplitude, which are responsible for the largest uncertainty in $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, have been calculated to NLLA[11]. The residual uncertainty in the charm amplitude is estimated to be $\sim 15\%$ which leads

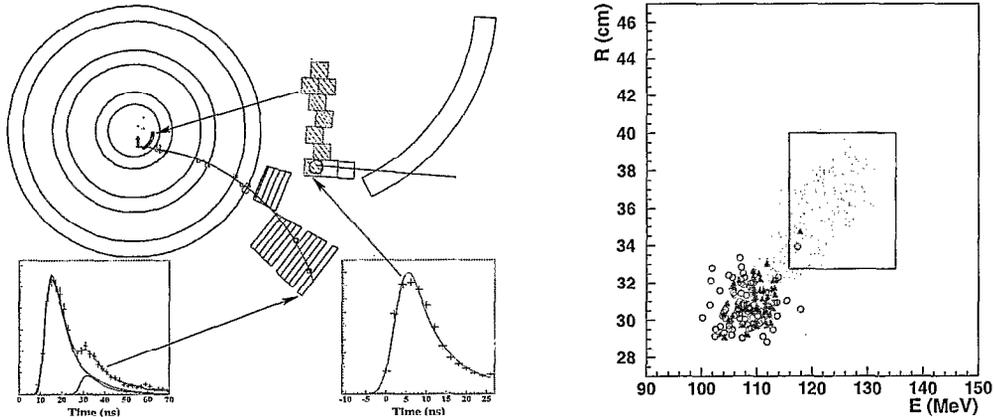


FIGURE 4. Left: new $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event. Right: Range vs energy of π^+ in the final sample. The circles are 1998 data and the triangles 1995-7 data. The events around $E = 108$ MeV are $K^+ \rightarrow \pi^+ \pi^0$ background. The simulated distribution of expected signal events is indicated by dots.

to only a $\sim 6\%$ uncertainty[23] in extracting $|V_{td}|$ from $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. Recently AGS E787 has seen evidence for a second event of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [24] (see Fig. 4), yielding a branching ratio $B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.57^{+1.75}_{-0.82}) \times 10^{-10}$. By comparison, a fit to the CKM phenomenology yields the expectation $(0.72 \pm 0.21) \times 10^{-10}$ [25]. It is notable that E787 has established methods to reduce the residual background to $\sim 10\%$ of the the signal branching ratio predicted by the SM.

A new experiment, AGS E949[26], based on an upgrade of the E787 detector, is about to begin its first physics run. Using the entire flux of the AGS for 6000 hours, it is designed to reach a sensitivity of $\sim 10^{-11}$ /event. In June 2001, Fermilab gave Stage 1 approval of an experiment, CKM[27], to extend the study of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ by yet another order of magnitude in sensitivity. This experiment, unlike all previous ones on this process, uses an in-flight rather than a stopping K^+ technique. This experiment is expected to start collecting data in 2007 or 2008.

Fig. 5 shows the history and expectations of progress in studying this decay.

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

$K_L \rightarrow \pi^0 \nu \bar{\nu}$ is the primary remaining target in the kaon system. Direct CP-violating to a very good approximation[10, 28], like $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ it has a hadronic matrix element that can be obtained from K_{e3} , but, unlike $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, it has no significant contribution from charm. As a result, the intrinsic theoretical uncertainty connecting $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ to the fundamental SM parameters is only about 2%. Note that $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is directly proportional to the square of $Im\lambda_t$ and that $Im\lambda_t = -\mathcal{J}/[\lambda(1 - \frac{\lambda^2}{2})]$ where \mathcal{J} is the Jarlskog invariant[29]. Thus a measurement of $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ determines the area of the unitarity triangles with a precision twice as good as that on $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ itself.

$B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ can be bounded indirectly by measurements of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ through a nearly model-independent relationship pointed out by Grossman and Nir[30].

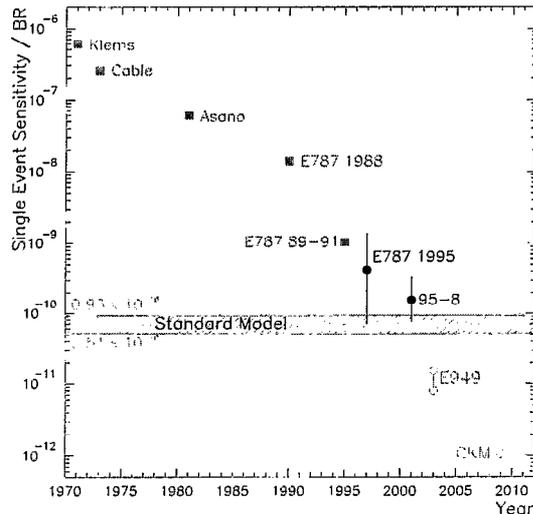


FIGURE 5. History and prospects for the study of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Points without error bars are single event sensitivities, those with error bars are measured branching ratio.

The application of this to the new E787 result yields $B(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 1.7 \times 10^{-9}$ at 90% CL. This is far tighter than the current direct experimental limit, 5.9×10^{-7} , obtained by KTeV[31]. To actually measure $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ at the SM level ($\sim 3 \times 10^{-11}$), one will need to improve on this by some five orders of magnitude. The KEK E391a experiment[32] does not quite propose to bridge this gap, but to achieve a sensitivity of $\sim 3 \times 10^{-10}$ /event, which would at least better the indirect limit by a factor five. It will serve as a test for a future much more sensitive experiment to be performed at the Japanese Hadron Facility. E391a features a carefully designed “pencil” beam, and a very high performance photon veto. The active photon detector is a CsI-pure crystal calorimeter. The entire rather compact apparatus will operate in vacuum. Beamline construction and tuning started in March 2000 and physics running is expected to begin in Fall, 2003.

The KOPIO experiment[33] at BNL takes a completely different approach, exploiting the intensity and flexibility of the AGS to make a high-flux, low-energy, microbunched K_L beam. The proposed experiment is shown in Fig. 6. The neutral beam will be extracted at $\sim 45^\circ$ to soften the K_L spectrum sufficiently to permit time-of-flight determination of the K_L velocity. The large production angle also softens the neutron spectrum so that the beam particles are by and large below threshold for the hadro-production of π^0 's. The beam region will be evacuated to 10^{-7} Torr to further minimize such production. With a 10m beam channel and this low energy beam, the contribution of hyperons to the background will be negligible. K_L decays from a ~ 3 m fiducial region will be accepted. Signal photons impinge on a $2 X_0$ thick preradiator capable of measuring their direction. An alternating drift chamber/scintillator plane structure will allow energy measurement as well. A high-precision shashlyk calorimeter downstream of the preradiator will complete the energy measurement. The photon directional information will allow the decay vertex position to be determined. Combined with the target position and

time of flight information, this provides a measurement of the K_L 3-momentum so that kinematic constraints as well as photon vetoing are available to suppress backgrounds. The leading expected background is $K_L \rightarrow \pi^0 \pi^0$, which is initially some eight orders of magnitude larger than the predicted signal. However since π^0 's from this background have a unique energy in the K_L center of mass, a very effective kinematic cut can be applied. This reduces the load on the photon veto system surrounding the decay region to the point where the hermetic veto techniques proven in E787 are sufficient. In fact most of the techniques necessary for KOPIO have been proven in previous experiments or in prototype tests. KOPIO aims to collect about 50 $K_L \rightarrow \pi^0 \nu \bar{\nu}$ events with a signal to background ratio of 2:1. This will permit η to be determined to $\sim 10\%$, given expected progress in measuring m_t and V_{cb} . KOPIO will run during the ~ 20 hours/day the AGS is not needed for injection into RHIC.

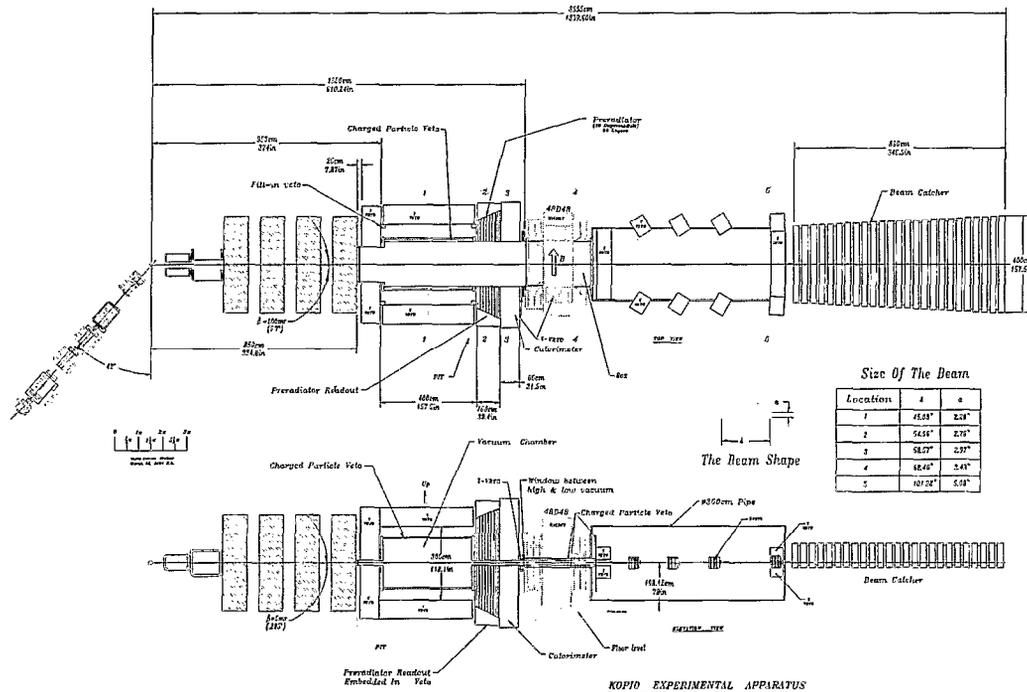


FIGURE 6. Layout of the KOPIO detector.

$$K_L \rightarrow \pi^0 \ell^+ \ell^-$$

These are reactions initially thought experimentally more tractable than $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Like $K_L \rightarrow \pi^0 \nu \bar{\nu}$, in the SM they are sensitive to $Im\lambda_t$, but in general they have different sensitivity to BSM effects[34]. Although their signatures are intrinsically superior to that of $K_L \rightarrow \pi^0 \nu \bar{\nu}$, they are subject to a background that has no analogue in the case of the latter: $K_L \rightarrow \gamma \ell^+ \ell^-$. This process, a radiative correction

to $K_L \rightarrow \gamma \ell^+ \ell^-$, occurs roughly 10^5 times more frequently than $K_L \rightarrow \pi^0 \ell^+ \ell^-$. Kinematic cuts are quite effective, but it is very difficult to improve the signal:background beyond about 1 : 10[35]. Both varieties of $K_L \rightarrow \gamma \gamma \ell^+ \ell^-$ have been observed, $B(K_L \rightarrow \gamma \gamma e^+ e^-)_{k_\gamma > 5 \text{ MeV}} = (5.84 \pm 0.15(\text{stat}) \pm 0.32(\text{syst})) \times 10^{-7}$ [36] and $B(K_L \rightarrow \gamma \gamma \mu^+ \mu^-)_{m_{\gamma\gamma} > 1 \text{ MeV}/c^2} = (10.4_{-5.9}^{+7.5}(\text{stat}) \pm 0.7(\text{syst})) \times 10^{-9}$ [37]; both agree with theoretical prediction. By comparison, in the SM $B^{\text{direct}}(K_L \rightarrow \pi^0 e^+ e^-)$ is predicted to be [38] $(4.3 \pm 2.1) \times 10^{-12}$ and $B^{\text{direct}}(K_L \rightarrow \pi^0 \mu^+ \mu^-)$ about five times smaller.

In addition to this background, there are two other issues that make the extraction of short-distance information from $K_L \rightarrow \pi^0 \ell^+ \ell^-$ problematical. First, there is an indirect CP-violating contribution from the K_1 component of K_L given by $|\epsilon|^2 \frac{\tau_{K_L}}{\tau_{K_S}} B(K_S \rightarrow \pi^0 e^+ e^-)$ which is of the same order of magnitude as the direct CP-violating piece². The exact size of this contribution will be predictable if and when $B(K_S \rightarrow \pi^0 e^+ e^-)$ is measured, hopefully by the upcoming NA48/1 experiment[39]. Second is yet another contribution of similar size mediated by $K_L \rightarrow \pi^0 \gamma \gamma$ which is *CP-conserving*. To some extent this contribution can be predicted from measurements of the branching ratio and kinematic distributions of $K_L \rightarrow \pi^0 \gamma \gamma$, and thousands of these events have been observed. However as indicated in Table 3, a new result from NA48[40] disagrees by nearly 3σ from the previous result from KTeV[41]. The change in the vector meson exchange contribution, characterized by the parameter a_V , reduces the predicted size of $B^{\text{CP-cons}}(K_L \rightarrow \pi^0 e^+ e^-)$ considerably[42] which is good news for the prospects of measuring $B^{\text{direct}}(K_L \rightarrow \pi^0 e^+ e^-)$. However the validity of the current technique for predicting $B^{\text{CP-cons}}(K_L \rightarrow \pi^0 e^+ e^-)$ from $K_L \rightarrow \pi^0 \gamma \gamma$ has recently been reexamined [43], and questions raised about the functions used to fit the spectrum and about the treatment of the dispersive contribution. Thus both the theoretical and experimental situations are quite unsettled at the moment. Depending on whose data and whose theory one uses, values from 0.25×10^{-12} to 7.3×10^{-12} are predicted for $B^{\text{CP-cons}}(K_L \rightarrow \pi^0 e^+ e^-)$.

TABLE 3. Results on $K_L \rightarrow \pi^0 \gamma \gamma$.

Exp.	$B(K_L \rightarrow \pi^0 \gamma \gamma) \times 10^6$	a_V	Ref.
KTeV	$1.68 \pm 0.07_{\text{stat}} \pm 0.08_{\text{syst}}$	$-0.72 \pm 0.05 \pm 0.06$	[41]
NA48	$1.36 \pm 0.03_{\text{stat}} \pm 0.03_{\text{syst}} \pm 0.03_{\text{norm}}$	$-0.46 \pm 0.03 \pm 0.03 \pm 0.02_{\text{theor}}$	[40]

The current experimental status of $K_L \rightarrow \pi^0 \ell^+ \ell^-$ is summarized in Table 4. A factor ~ 2.5 more data is expected from the KTeV 1999 run, but as can be seen from the table, background is already starting to be observed at a sensitivity roughly 100 times short of the expected signal level.

TABLE 4. Results on $K_L \rightarrow \pi^0 \ell^+ \ell^-$.

Mode	90% CL upper limit	Est. bkgnd.	Obs. evts.	Ref.
$K_L \rightarrow \pi^0 e^+ e^-$	5.1×10^{-10}	1.06 ± 0.41	2	[44]
$K_L \rightarrow \pi^0 \mu^+ \mu^-$	3.8×10^{-10}	0.87 ± 0.15	2	[45]

² There is also an interference term between the indirect and direct CP-violating amplitudes.

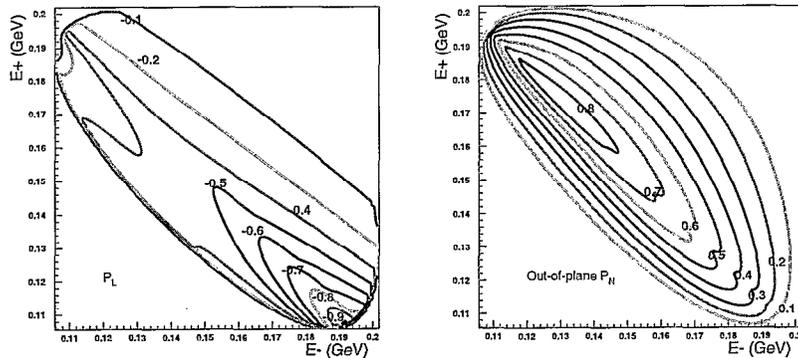


FIGURE 7. μ^+ polarizations in $K_L \rightarrow \pi^0 \mu^+ \mu^-$, plotted against the muon cm energies. Left: longitudinal polarization. Right: out-of-plane polarization.

To make a useful measurement under these conditions will require markedly increased statistics on the signal and both theoretical and experimental advances in the ancillary modes $K_L \rightarrow \pi^0 \gamma \gamma$ and $K_S \rightarrow \pi^0 e^+ e^-$. Various approaches for mitigating these problems have been suggested over the years including studies of the Dalitz Plot [46], the time development [47], or both [48]. However an innovative approach has recently been suggested [49] in which muon polarization in $K_L \rightarrow \pi^0 \mu^+ \mu^-$ as well as kinematic distributions are exploited. The μ^+ longitudinal polarization is proportional to the direct CP-violating amplitude, whereas the energy asymmetry and the out-of-plane polarization depend on both indirect and direct CP violating amplitudes. As shown in Fig. 7, the polarizations involved turn out to be extremely large so that enormous numbers of events may not be required.

An alternative parameterization

Although it is customary to write the branching ratios and other observables of the one-loop processes in terms of the Wolfenstein parameterization of the CKM matrix, this parameterization is not really natural to the kaon system, and puts results from this system at a certain disadvantage in comparisons with those from the B system. To extract information on ρ and η , for example, it is necessary to divide the physical measurements by $\lambda^8 A^4$, thereby introducing “external” contributions to the uncertainty of $8\sigma_\lambda$ and $4\sigma_A$. One can avoid this by resorting to expressions for the branching ratios in terms of the quantity λ_t . Since as noted above, the imaginary part of this quantity determines the area of all unitarity triangles, it is no less fundamental than ρ and η .

The formulae for the branching ratios of three of the decays discussed above are:

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \xi [(\lambda_c \bar{X} + \text{Re}(\lambda_t) X_t)^2 + (\text{Im}(\lambda_t) X_t)^2] \quad (1)$$

$$B^{SD}(K_L \rightarrow \mu^+ \mu^-) = \xi' [\text{Re}(\lambda_c) Y_{NL} + \text{Re}(\lambda_t) Y(x_t)]^2 \quad (2)$$

$$B(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \xi'' (Im(\lambda_t) X_t)^2 \quad (3)$$

where

$$\xi \equiv \frac{3r_{K^+} \alpha^2 B_{K^+ e3}}{V_{us}^2 2\pi^2 \sin^4 \theta_W} = 1.55 \times 10^{-4} \quad (4)$$

$$\xi' \equiv \frac{\tau_{K_L} \alpha^2 B_{K^+ \mu \nu}}{\tau_{K^+} V_{us}^2 \pi^2 \sin^4 \theta_W} = 6.32 \times 10^{-3} \quad (5)$$

$$\xi'' \equiv \frac{3r_{K_L} \tau_{K_L} \alpha^2 B_{K^+ e3}}{\tau_{K^+} V_{us}^2 2\pi^2 \sin^4 \theta_W} = 6.77 \times 10^{-4} \quad (6)$$

and to a good approximation the Inami-Lim [50] functions $X_t = 1.56(m_t/170\text{GeV})^{1.15}$, $Y_t = 1.02(m_t/170\text{GeV})^{1.56}$. The quantities $r_{K^+} = 0.901$ and $r_{K_L} = 0.944$ are isospin correction factors that relate the hadronic matrix elements of the $K \rightarrow \pi \nu \bar{\nu}$ processes to that of $K^+ \rightarrow \pi^0 e^+ \nu$ [22]. The terms \bar{X} and Y_{NL} are the Inami-Lim functions for the charm contributions which, after correction to NLLA, are known to about 15%.

Fig. 8 shows the 90% CL constraints currently available from $K_L \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. To extract a limit from the former we adopt the value for the maximum long distance dispersive contribution from Ref. [21]. Also shown is the region in the λ_t plane bounded by the CKM fit mentioned above [16]. The two kinds of information are clearly consistent at the moment. Note, however, that if the current central value of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ should hold through E949, and expected progress is made in the B sector, this agreement could prove short-lived, as shown on the left of Fig. 9. When, eventually, we have 10-15% measurements of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$, comparison of K and B results will become a critical test of the SM. Fig. 9 right illustrates a scenario in which such a failure is evident.

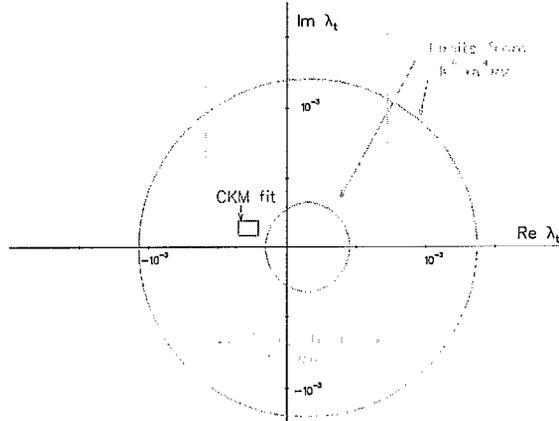


FIGURE 8. Comparison of 90% CL constraints from current data on rare kaon decays with 95% CL constraints from a typical unitarity fit (based on Ref. [16]). The allowed region from kaon decays lies between the two circles and within the outer two vertical lines.

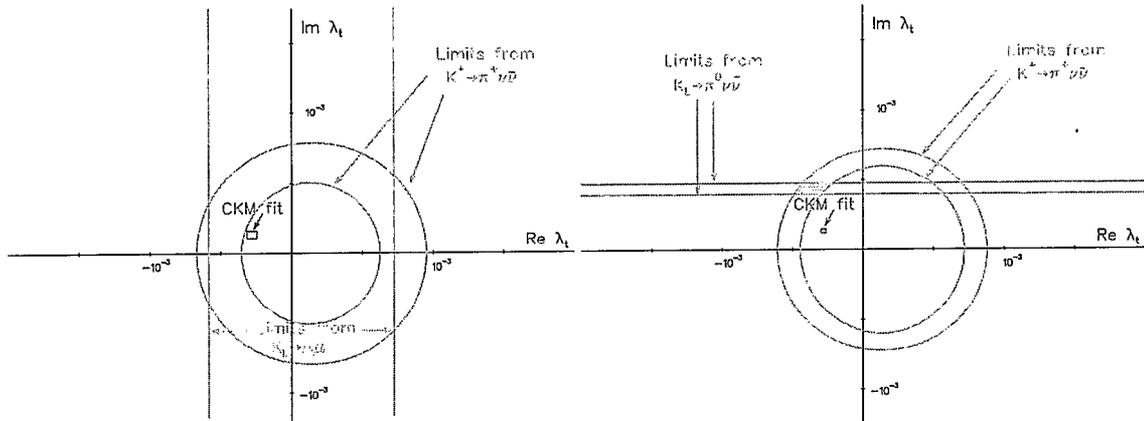


FIGURE 9. Left: Similar plot to Fig. 8 after $10^{-11}/\text{event}$ $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ experiment. Assumes central value of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ stays the same and also that CKM fit contours and precision on m_t are improved by a factor 2. Right: Similar plot for possible scenario after 10% measurements of $|\lambda_t|$ and $Im(\lambda_t)$. Further improvements in CKM parameters and m_t assumed.

CONCLUSIONS

The success of lepton flavor violation experiments in reaching sensitivities corresponding to mass scales of well over 100 TeV has helped kill most models predicting accessible LFV in kaon decay. The most popular varieties of SUSY predict LFV at levels far beyond the current experimental state of the art [51]. Thus new dedicated experiments in this area are unlikely in the near future.

The existing precision measurement of $K_L \rightarrow \mu^+ \mu^-$ will be very useful if theorists can make enough progress on calculating the dispersive long-distance amplitude, perhaps helped by experimental progress in $K_L \rightarrow \gamma \ell^+ \ell^-$, $K_L \rightarrow 4 \text{ leptons}$, etc. The exploitation of $K_L \rightarrow \mu^+ \mu^-$ would also be aided by higher precision measurements of some of the normalizing reactions, such as $K_L \rightarrow \gamma \gamma$.

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ will clearly be further exploited. Two coordinated initiatives are devoted to this: a $10^{-11}/\text{event}$ experiment (E949) just underway at the BNL AGS and a $10^{-12}/\text{event}$ experiment (CKM) recently approved for the FNAL Main Injector. The first dedicated experiment to seek $K_L \rightarrow \pi^0 \nu \bar{\nu}$ (E391a) is proceeding and an experiment (KOPPIO) at the AGS with the goal of making a $\sim 10\%$ measurement of $Im(\lambda_t)$ is approved and in R&D.

Measurements of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ can determine an alternative unitarity triangle that will offer a critical comparison with results from the B system. If new physics is in play in the flavor sector, the two triangles will almost certainly disagree.

ACKNOWLEDGMENTS

I thank D. Bryman, M. Diwan, F. Gabbiani, D. Jaffe, S. Kettell, G. Valencia, and M. Zeller for useful discussions, access to results, and other materials. This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886.

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