The Color Glass Condensate

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The Color Glass Condensate is a state of high density gluonic matter which controls the high energy limit of hadronic interactions. Its properties are important for the initial conditions for matter produced at RHIC.

1. Introduction

It is a real delight to be back in Bielefeld 20 years after the historic first meeting on the properties of Satistical QCD organized by Helmut Satz. In this time, we have seen the theoretical ideas first discussed at this meeting tested in the experiments at CERN and at RHIC. The RHIC machine itself would almost certainly not exist had it not been for the intellectual excitement generated at the Bielefeld meeting.

On a personal note, I met a large fraction of the people with whom I have collaborated in the last twenty years, and many more with whom I have established life long friendships. The meeting strongly influenced the direction of my research throughout my career. I believe this is true for many of the other participants.

During this time, Helmut has built a very strong group here, and seeded the development of groups around the world. He has also been very influential in the development of the experimental programs at CERN, the AGS, RHIC and the future program at LHC.

The topic I discuss will be perhaps a little off to the side from the discussion of the quark gluon plasma and how it may appear in heavy ion collisions. It is more about the early stages of such collisions and the wavefunctions of the ultrarelativistic nuclei themselves. We shall see that to describe these wavefunctions, we shall introduce a form of matter, and this matter has properties similar to those of a quark-gluon plasma, but in some fundamental ways is different.

Let me begin with some obvious truisms: QCD is the correct theory of hadronic physics. It has been tested in various experiments. For high energy short distance phenomena, perturbative QCD computations successfully confront experiment. In lattice Monte-Carlo computations, one gets a successful semi-quantitative description of hadronic spectra, and perhaps in the not too distant future one will obtain precise quantitative agreement.

At present, however, all analytic computations and all precise QCD tests are limited to a small class of problems which correspond to short distance physics, or to semi-quantitative comparisons with the results of lattice gauge theory numerical computations. For the short distance phenomena, there is some characteristic energy transfer scale E, and one uses asymptotic freedom,

$$\alpha_S(E) \to 0 \qquad (1)$$

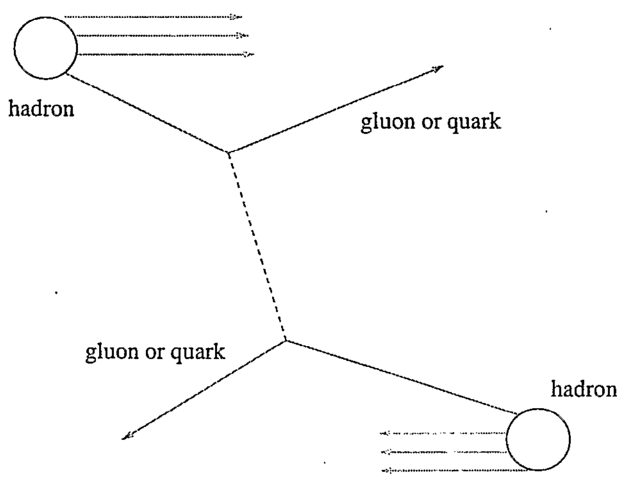

Figure 1. Hadron-hadron scattering to produce a pair of jets.

as $E \to \infty$ For example, in Fig. 1, two hadrons collide to make a pair of jets. If the transverse momenta of the jets is large, the strong coupling strength which controls this production is evaluated at the p_T of the jet. If $p_T >> \Lambda_{QCD}$, then the coupling is weak and this process can be computed in perturbation theory. QCD has also been extensively tested in deep inelastic scattering. In Fig. 2, an electron exchanges a virtual photon with a hadronic target. If the virtual photon momentum transfer Q is large, then one can use weak coupling methods.

One question which we might ask is whether there are non-perturbative "simple phenomena" which arise from QCD which are worthy of further effort. The questions I would ask before I would become interested in understanding such phenomena are

- Is the phenomenon simple in structure?
- Is the phenomena pervasive?
- Is it reasonably plausible that one can understand the phenomena from first principles, and compute how it would appear in nature?

I will argue that *gross* or *typical* processes in QCD, which by their very nature are pervasive, appear to follow simple patterns. The main content of this first lecture is to show some of these processes, and pose some simple questions about their nature which we do not yet understand.

My goal is to convince you that much of these average phenomena of strong interactions at extremely high energies is controlled by a new form of hadronic matter, a dense condensate of gluons. This is called the Color Glass Condensate since

- Color: The gluons are colored.
- Glass: We shall see that the fields associated with the glass evolve very slowly relative to natural time scales, and are disordered. This is like a glass which is

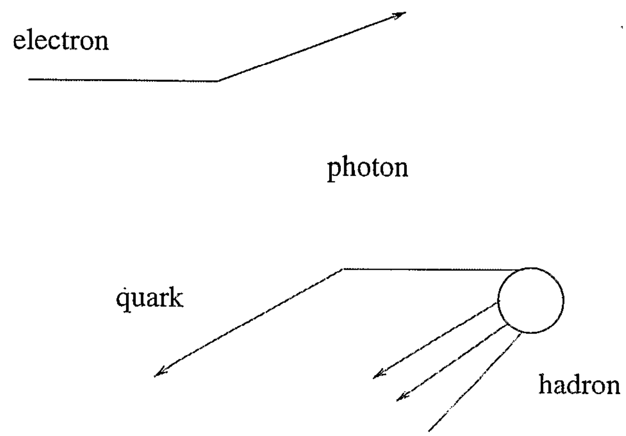

Figure 2. Deep inelastic scattering of an electron on a hadron.

disordered and is a liquid on long time scales but seems to be a solid on short time scales.

- Condensate: There is a very high density of massless gluons. These gluons can be packed until their phase space density is so high that interactions prevent more gluon occupation. This forces at increasingly high density the gluons to occupy higher momenta, and the coupling becomes weak. The density saturates at $dN/d^2p_Td^2r_T \sim 1/\alpha_s >> 1$, and is a condensate.

In these lectures, I will try to explain why the above is very plausible.

2. Total Cross Sections at Asymptotic Energy

Computing total cross sections as $E \to \infty$ is one of the great unsolved problems of QCD. Unlike for processes which are computed in perturbation theory, it is not required that any energy transfer become large as the total collision energy $E \to \infty$. Computing a total cross section for hadronic scattering therefore appears to be intrinsically non-perturbative. In the 60's and early 70's, Regge theory was extensively developed in an attempt to understand the total cross section. The results of this analysis were to my mind inconclusive, and certainly can not be claimed to be a first principles understanding from QCD.

The total cross section for pp and $\bar{p}p$ collisions is shown in Fig. 3. Typically, it is assumed that the total cross section grows as $ln^2 E$ as $E \to \infty$. This is the so called Froisart bound which corresponds to the maximal growth allowed by unitarity of the S matrix. Is this correct? Is the coefficient of $ln^2 E$ universal for all hadronic precesses? Why is the unitarity limit saturated? Can we understand the total cross section from first principles in QCD? Is it understandable in weakly coupled QCD, or is it an intrinsically non-perturbative phenomenon?

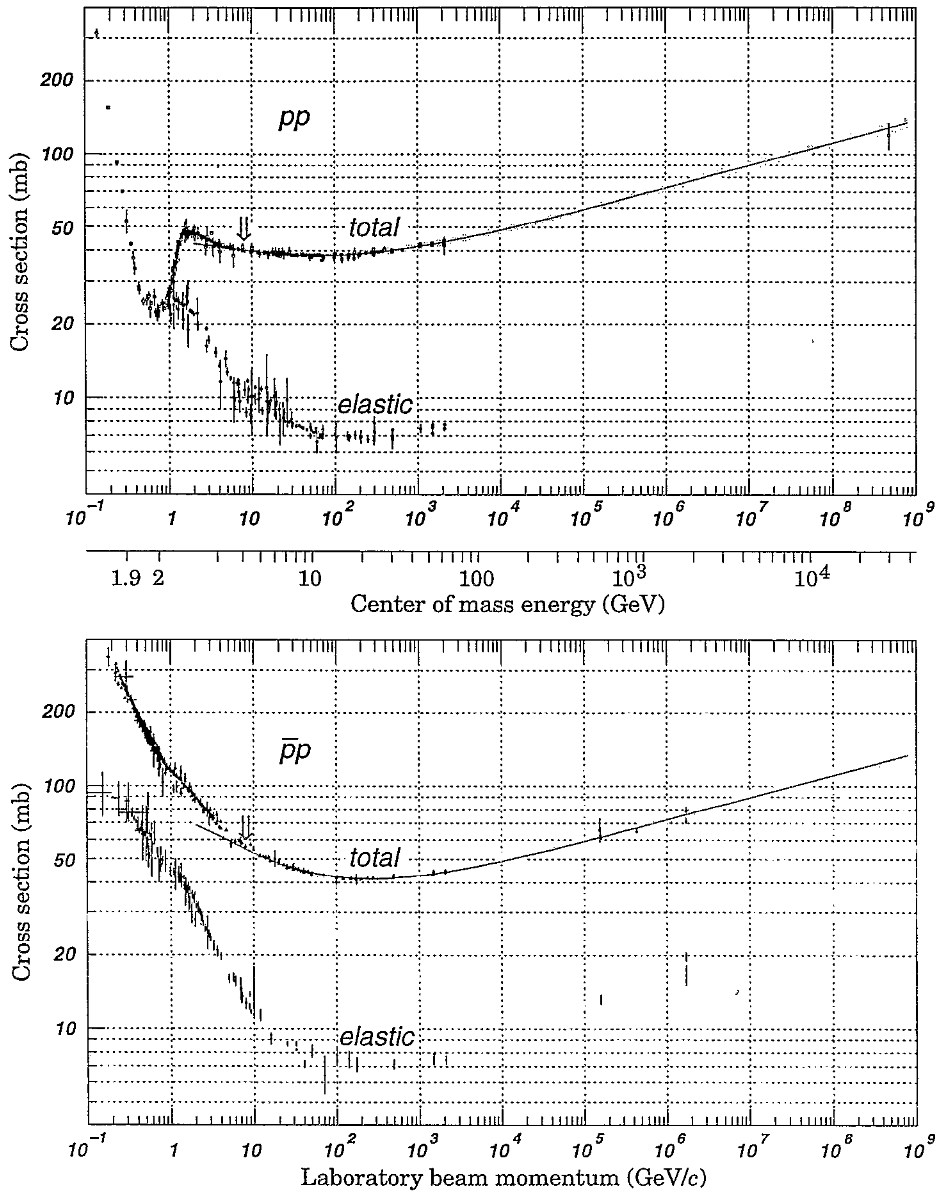

Figure 3. The cross sections for *pp* and $p\overline{p}$ scattering.

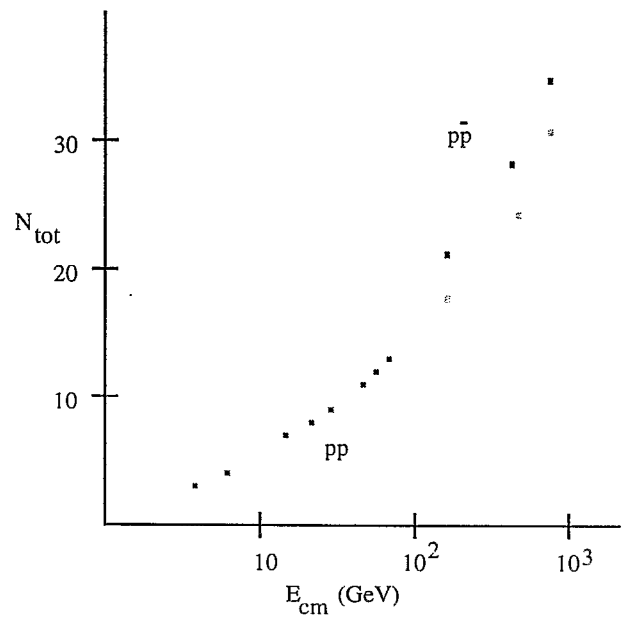

Figure 4. The total multiplicity in pp and $p\overline{p}$ collisions.

3. How Are Particles Produced in High Energy Collisions?

In Fig. 4 , I plot the multiplicity of produced particles in pp and in $\overline{p}p$ collisions. The last six points correspond to the $\overline{p}p$ collisions. The three upper points are the multiplicity in $p\overline{p}$ collisions, and the bottom three have the mutliplicity at zero energy subtracted. The remaining points correspond to pp. Notice that the pp points and those for $p\overline{p}$ with zero energy multiplicity subtracted fall on the same curve. The implication is that whatever is causing the increase in multiplicity in these collisions may be from the same mechanism. Can we compute $N(E)$, the total multiplicity of produced particles as a function of energy?

4. Some Useful Variables

At this point it is useful to develop some mathematical tools. I will introduce kinematic variables: light cone coordinates. Let the light cone longitudinal momenta be

$$p^{\pm} = \frac{1}{\sqrt{2}}(E \pm p_z) \tag{2}$$

Note that the invariant dot product

$$p \cdot q = p_t \cdot q_t - p^+q^- - p^-q^+ \tag{3}$$

and that

$$p^+p^- = \frac{1}{2}(E^2 - p_z^2) = \frac{1}{2}(p_T^2 + m^2) = \frac{1}{2}m_T^2 \tag{4}$$

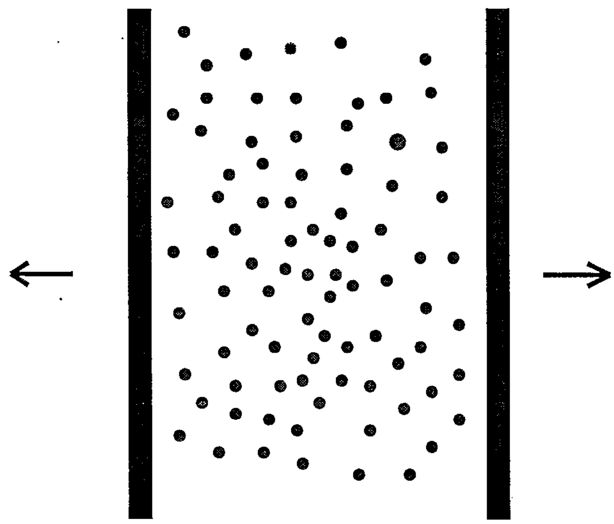

Figure 5. A hadron-hadron collision. The produced particles are shown as red circles.

This equation defines the transverse mass m_T. (Please note that my metric is the negative of that conventionally used in particle physics.)

Consider a collision in the center of mass frame as shown in Fig. 5. In this figure, we have assumed that the colliding particles are large compared to the size of the produced particles. This is true for nuclei, or if the typical transverse momenta of the produced particles is large compared to Λ_{QCD}, since the corresponding size will be much smaller than a Fermi. We have also assumed that the colliding particles have an energy which is large enough so that they pass through one another and produce mesons in their wake. This is known to happen experimentally: the particles which carry the quantum numbers of the colliding particles typically lose only some finite fraction of their momenta in the collision.

The right moving particle which initiates the collision shown in Fig. 5 has $p_1^+ \sim \sqrt{2} \mid p_z \mid$ and $p_1^- \sim \frac{1}{2\sqrt{2}} m_T^2 / \mid p_z \mid$. For the colliding particles $m_T = m_{projectile}$, that is because the transverse momentum is zero, the transverse mass equals the particle mass. For particle 2, we have $p_2^+ = p_1^-$ and $p_2^- = p_1^+$.

If we define the Feynman x of a produced pion as

$$x = p_\pi^+ / p_1^+ \tag{5}$$

then $0 \leq x \leq 1$. (This definition agrees with Feynman's original one if the energy of a particle in the center of mass frame is large and the momentum is positive. We will use this definition as a generalization of the original one of Feynman since it is invariant under longitudinal Lorentz boosts.) The rapidity of a pion is defined to be

$$y = \frac{1}{2} ln(p_\pi^+ / p_\pi^-) = \frac{1}{2} ln(2p^{+2}/m_T^2) \tag{6}$$

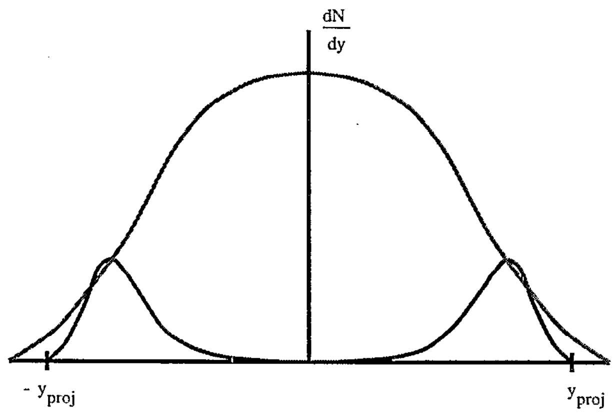

Figure 6. The rapidity distribution of particles produced in a hadronic collision.

For pions, the transverse mass includes the transverse momentum of the pion.

The pion rapidity is always in the range $-y_{CM} \leq y \leq y_{CM}$ where $y_{CM} = ln(p^+/m_{projectile})$ All the pions are produced in a distribution of rapidities within this range.

A distribution of produced particles in a hadronic collision is shown in Fig. 6. The leading particles are shown in blue and are clustered around the projectile and target rapidities. For example, in a heavy ion collision, this is where the nucleons would be. In red, the distribution of produced mesons is shown.

These definitions are useful, among other reasons, because of their simple properties under longitudinal Lorentz boosts: $p^{\pm} \rightarrow \kappa^{\pm 1} p^{\pm}$ where κ is a constant. Under boosts, the rapidity just changes by a constant.

The distribution of mesons, largely pions, shown in Fig. 4. are conveniently thought about in the center of mass frame. Here we imagine the positive rapidity mesons as somehow related to the right moving particle and the negative rapidity particles as related to the left moving particles. We define $x = p^+/p^+_{projectile}$ and $x' = p^-/p^-_{projectile}$ and use x for positive rapidity pions and x' for negative rapidity pions.

Several theoretical issues arise in multiparticle production. Can we compute dN/dy? or even dN/dy at $y = 0$? How does the average transverse momentum of produced particles $< p_T >$ behave with energy? What is the ratio of produced strange/nonstrange mesons, and corresponding ratios of charm, top, bottom etc at $y = 0$ as the center of mass energy approaches infinity?

Does multiparticle production as $E \rightarrow \infty$ at $y = 0$ become simple, understandable and computable?

There is a remarkable feature of rapidity distributions of produced hadrons, which we shall refer to as Feynman scaling. If we plot rapidity distributions of produced hadrons at different energies, then as function of the distance from the fragmentation region, the rapidity distributions are to a good approximation independent of energy. This is illustrated in Fig. 7. This means that as we go to higher and higher energies, the new physics is associated with the additional degrees of freedom at small

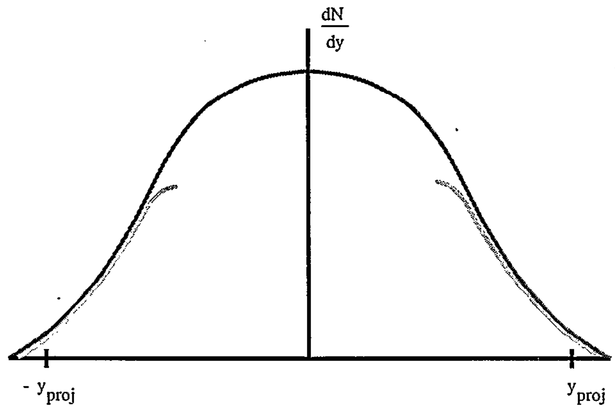

Figure 7. Feynman scaling of rapidity distributions. The two different colors correspond to rapidity distributions at different energies.

rapidities in the center of mass frame (small-x degrees of freedom). The large x degrees of freedom do not change much. This suggests that there may be some sort of renormalization group description in rapidity where the degrees of freedom at larger x are held fixed as we go to smaller values of x. We shall see that in fact these large x degrees of freedom act as sources for the small x degrees of freedom, and the renormalization group is generated by integrating out low x degrees of freedom to generate these sources.

5. Deep Inelastic Scattering

In Fig. 2, deep inelastic scattering is shown. Here an electron emits a virtual photon which scatters from a quark in a hadron. The momentum and energy transfer of the electron is measured, and the results of the break up are not. In these lectures, I do not have sufficient time to develop the theory of deep inelastic scattering. Suffice it to say, that this measurement is sufficient at large momenta transfer Q^2 to measure the distributions of quarks in a hadron.

To describe the quark distributions, it is convenient to work in a reference frame where the hadron has a large longitudinal momentum p^+_{hadron}. The corresponding light cone momentum of the constituent is $p^+_{constituent}$. We define $x = p^+_{constituent}/p^+_{hadron}$. This x variable is equal to the Bjorken x variable, which can be defined in a frame independent way. In this frame independent definition, $x = Q^2/2p \cdot Q$ where p is the momentum of the hadronic target and Q is the momentum of the virtual photon. The cross section which one extracts in deep inelastic scattering can be related to the distributions of quarks inside a hadron, dN/dx.

It is useful to think about the distributions as a function of rapidity. We define this for deep inelastic scattering as

$$y = y_{hadron} - ln(1/x) \qquad (7)$$

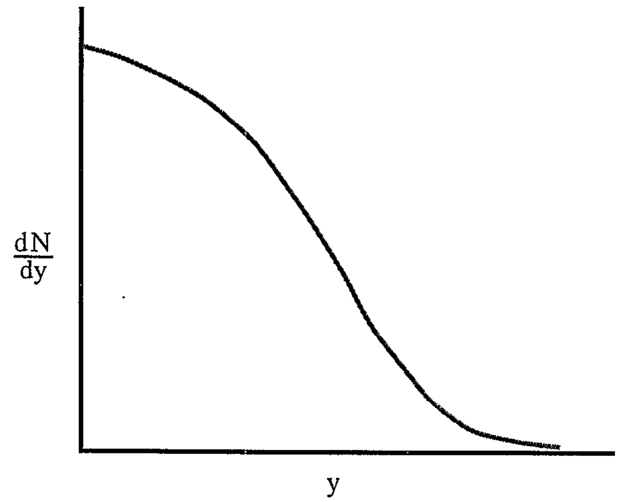

Figure 8. The rapidity distribution of gluons inside of a hadron.

and the invariant rapidity distribution as

$$dN/dy = xdN/dx \tag{8}$$

In Fig. 8, a typical dN/dy distribution for constituent gluons of a hadron is shown. This plot is similar to the rapidity distribution of produced particles in deep inelastic scattering. The main difference is that we have only half of the plot, corresponding to the left moving hadron in a collision in the center of mass frame.

We shall later argue that there is in fact a relationship between the structure functions as measured in deep inelastic scattering and the rapidity distributions for particle production. We will argue that the gluon distribution function is in fact proportional to the pion rapidity distribution.

The small x problem is that in experiments at Hera, the rapidity distribution function for quarks grows as the rapidity difference between the quark and the hadron grows. This growth appears to be more rapid than simply $\mid y_{proj} - y \mid$ or $(y_{proj} - y)^2$, and various theoretical models based on the original considerations of Lipatov and colleagues suggest it may grow as an exponential in $\mid y_{proj} - y \mid$.[1] (Consistency of the BFKL approach with the more established DGLAP evolution equations remains an outstanding theoretical problem.[2]) If the rapidity distribution grew at most as y^2, then there would be no small x problem. We shall try to explain the reasons for this later in this lecture.

In Fig. 9, the Zeus data for the gluon structure function is shown.[3] I have plotted the structure function for $Q^2 = 5\ GeV^2$, $20\ GeV^2$ and $200\ GeV^2$. The structure function depends upon the resolution of the probe, that is Q^2. Note the rise of $xg(x)$ at small x. this is the small x problem. If one had plotted the total multiplicity of produced particles in pp and $\overline{p}p$ collisions on the same plot, one would have found rough agreement in the shape of the curves. Here I would use $y = log(E_{cm}/1\ GeV)$ for the pion production data.

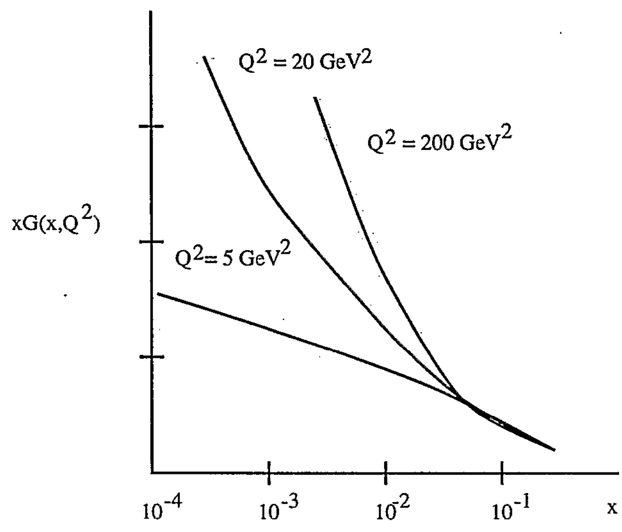

Figure 9. The Zeus data for the gluon structure functions.

This is approximately the maximal value of rapidity difference between centrally produced pions and the projectile rapidity. The total multiplicity would be rescaled so that at small x, it matches the gluon structure functions. This demonstrates the qualitative similarity between the gluon structure function and the total multiplicity.

Why is the small x rise in the gluon distribution a problem? Consider Fig. 10, where we view hadron head on.[4]-[5] The constituents are the valence quarks, gluons and sea quarks shown as colored circles. As we add more and more constituents, the hadron becomes more and more crowded. If we were to try to measure these constituents with say an elementary photon probe, as we do in deep inelastic scattering, we might expect that the hadron would become so crowded that we could not ignore the shadowing effects of constituents as we make the measurement. (Shadowing means that some of the partons are obscured by virtue of having another parton in front of them. For hard spheres, for example, this would result in a decrease of the scattering cross section relative to what is expected from incoherent independent scattering.)

In fact, in deep inelastic scattering, we are measuring the cross section for a virtual photon γ^* and a hadron, $\sigma_{\gamma^* hadron}$. Making x smaller corresponds to increasing the energy of the interaction (at fixed Q^2). An exponential growth in the rapidity corresponds to power law growth in $1/x$, which in turn implies power law growth with energy. This growth, if it continues forever, violates unitarity. The Froissart bound will allow at most $ln^2(1/x)$. (The Froissart bound is a limit on how rapidly a total cross section can rise. It follows from the unitarity of the scattering matrix.)

We shall later argue that in fact the distribution functions at fixed Q^2 do in fact saturate and cease growing so rapidly at high energy. The total number of gluons however demands a resolution scale, and we will see that the natural intrinsic scale is growing at smaller values of x, so that effectively, the total number of gluons within this intrinsic scale is

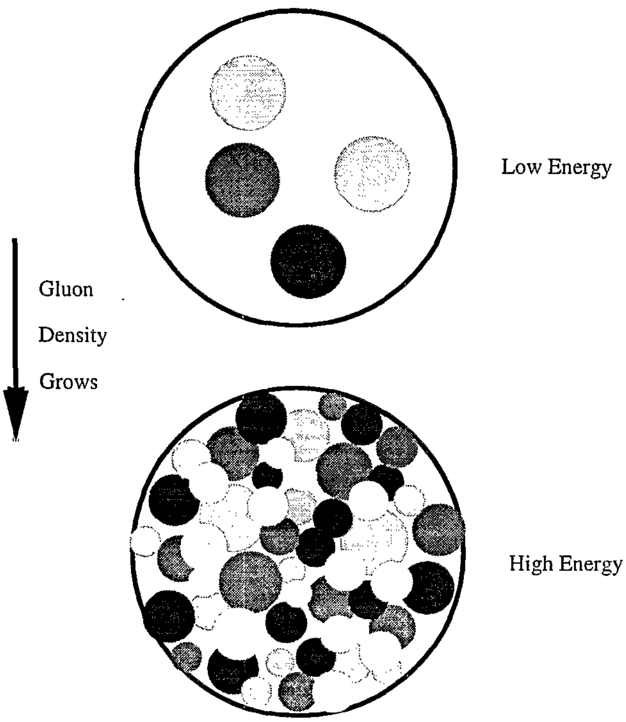

Figure 10. Saturation of gluons in a hadron. A view of a hadron head on as x decreases.

always increasing. The quantity

$$\Lambda^2 = \frac{1}{\pi R^2}\frac{dN}{dy} \tag{9}$$

defines this intrinsic scale. Here πR^2 is the cross section for hadronic scattering from the hadron. For a nucleus, this is well defined. For a hadron, this is less certain, but certainly if the wavelengths of probes are small compared to R, this should be well defined. If

$$\Lambda^2 >> \Lambda^2_{QCD} \tag{10}$$

as the Hera data suggests, then we are dealing with weakly coupled QCD since $\alpha_S(\Lambda) << 1$.

Even though QCD may be weakly coupled at small x, that does not mean the physics is perturbative. There are many examples of nonperturbative physics at weak coupling. An example is instantons in electroweak theory, which lead to the violation of baryon number. Another example is the atomic physics of highly charged nuclei. The electron propagates in the background of a strong nuclear Coulomb field, but on the other hand, the theory is weakly coupled and there is a systematic weak coupling expansion which allows for computation of the properties of high Z (Z is the charge of the nucleus) atoms.

We call this assortment of gluons a Color Glass Condensate. The name follows from the fact that the gluons are colored, and we have seen that they are very dense. For massless particles we expect that the high density limit will be a Bose condensate. The phase space density will be limited by repulsive gluon interactions, and be of order $1/\alpha_s >> 1$. The glass nature follows because these fields are produced by partons at higher rapidity, and in the center of mass frame, they are Lorentz time dilated. Therefore the induced fields at smaller rapidity evolve slowly compared to natural time scales. These fields are also disordered. These two properties are similar to that of a glass which is a disrodered material which is a liquid on long time scales and a solid on short ones.

If the theory is local in rapidity, then the only parameter which can determine the physics at that rapidity is Λ^2. Locality in rapidity means that there are not long range correlations in the hadronic wavefunction as a function of rapidity. In pion production, it is known that except for overall global conserved quantities such as energy and total charge, such correlations are of short range. Note that if only Λ^2 determines the physics, then in an approximately scale invariant theory such as QCD, a typical transverse momentum of a constituent will also be of order Λ^2. If $\Lambda^2 >> 1/R^2$, where R is the radius of the hadron, then the finite size of the hadron becomes irrelevant. Therefore at small enough x, all hadrons become the same. The physics should only be controlled by Λ^2.

There should therefore be some equivalence between nuclei and say protons. When their Λ^2 values are the same, their physics should be the same. We can take an empirical parameterization of the gluon structure functions as

$$\frac{1}{\pi R^2}\frac{dN}{dy} \sim \frac{A^{1/3}}{x^\delta} \tag{11}$$

where $\delta \sim .2 - .3$. This suggests that there should be the following correspondences:

- RHIC with nuclei $\sim$ Hera with protons
- LHC with nuclei $\sim$ Hera with nuclei

Estimates of the parameter Λ for nuclei at RHIC energies give $\sim 1 - 2$ Gev, and at LHC $2 - 3$ Gev.

Since the physics of high gluon density is weak coupling we have the hope that we might be able to do a first principle calculation of

- the gluon distribution function
- the quark and heavy quark distribution functions
- the intrinsic p_T distributions quarks and gluons

We can also suggest a simple escape from unitarity arguments which suggest that the gluon distribution function must not grow at arbitrarily small x. The point is that at smaller x, we have larger Λ and correspondingly larger p_T. A typical parton added to the hadron has a size of order $1/p_T$. Therefore although we are increasing the number of gluons, we do it by adding in more gluons of smaller and smaller size. A probe of size resolution $\Delta x \geq 1/p_T$ at fixed Q will not see partons smaller than this resolution size. They therefore do not contribute to the fixed Q^2 cross section, and there is no contradiction with unitarity.

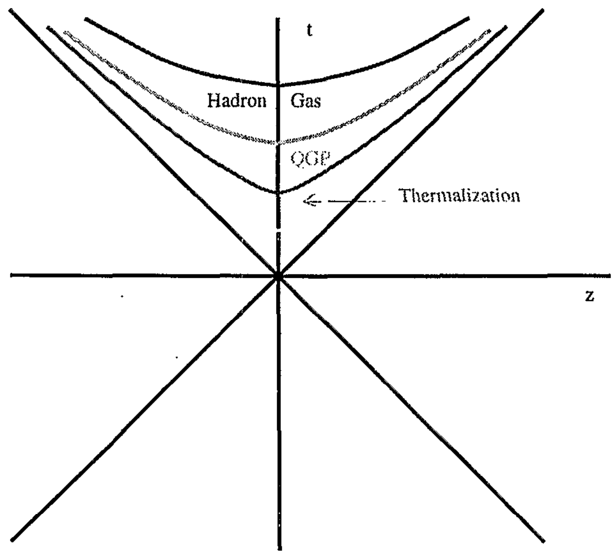

Figure 11. A space-time figure for ultrarelativistic heavy ion collisions.

6. Heavy Ion Collisions

In Fig. 11, the standard lightcone cartoon of heavy ion collisions is shown.[6] To understand the figure, imagine we have two Lorentz contracted nuclei approaching one another at the speed of light. Since they are well localized, they can be thought of as sitting at $x^{\pm} = 0$, that is along the light cone, for $t < 0$. At $x^{\pm} = 0$, the nuclei collide. To analyze this problem for $t \geq 0$, it is convenient to introduce a time variable which is Lorentz covariant under longitudinal boosts

$$\tau = \sqrt{t^2 - z^2} \tag{12}$$

and a space-time rapidity variable

$$\eta = \frac{1}{2} ln \left(\frac{t-z}{t+z} \right) \tag{13}$$

For free streaming particles

$$z = vt = \frac{p_z}{E} t \tag{14}$$

we see that the space-time rapidity equals the momentum space rapidity

$$\eta = y \tag{15}$$

If we have distributions of particles which are slowly varying in rapidity, it should be a good approximation to take the distributions to be rapidity invariant. This should be valid at very high energies in the central region. By the correspondence between space-time and momentum space rapidity, it is plausible therefore to assume that distributions are independent of η. Therefore distributions are the same on lines of constant τ, which is as shown in Fig. 11. At $z = 0$, $\tau = t$, so that τ is a longitudinally Lorentz invariant time variable.

We expect that at very late times, we have a free streaming gas of hadrons. These are the hadrons which eventually arrive at our detector. At some earlier time, these particles decouple from a dense gas of strongly interacting hadrons. As we proceed earlier in time, at some time there is a transition between a gas of hadrons and a plasma of quarks and gluons. This may be through a first order phase transition where the system might exist in a mixed phase for some length of time, or perhaps there is a continuous change in the properties of the system

At some earlier time, the quarks and gluons of the quark-gluon plasma are formed. This is at RHIC energies, a time of the order of a Fermi, perhaps as small as .1 *Fermi*. As they form, the particles scatter from one another, and this can be described using the methods of transport theory. At some later time they have thermalized, and the system can be approximately described using the methods of perfect fluid hydrodynamics.

In the time between that for which the quarks and gluons have been formed and $\tau = 0$, the particles are being formed. This is where the initial conditions for a hydrodynamic description are made.

In various levels of sophistication, one can compute the properties of matter made in heavy ion collisions at times later than the formation time. The problems are understood in principle for $\tau \geq \tau_{formation}$ if perhaps not in fact. Very little is known about the initial conditions.

In principal, understanding the initial conditions should be the simplest part of the problem. At the initial time, the degrees of freedom are most energetic and therefore one has the best chance to understand them using weak coupling methods in QCD.

There are two separate classes of problems one has to understand for the initial conditions. First the two nuclei which are colliding are in single quantum mechanical states. Therefore for some early time, the degrees of freedom must be quantum mechanical. This means that

$$\Delta z \Delta p_z \geq 1 \tag{16}$$

Therefore classical transport theory cannot describe the particle down to $\tau = 0$ since classical transport theory assumes we know a distribution function $f(\vec{p}, \vec{x}, t)$, which is a simultaneous function of momenta and coordinates. This can also be understood as a consequence of entropy. An initial quantum state has zero entropy. Once one describes things by classical distribution functions, entropy has been produced. Where did it come from?

Another problem which must be understood is classical charge coherence. At very early time, we have a tremendously large number of particles packed into a longitudinal size scale of less than a fermi. This is due to the Lorentz contraction of the nuclei. We know that the particles cannot interact incoherently. For example, if we measure the field due to two opposite charge at a distance scale r large compared to their separation, we know the field falls as $1/r^2$, not $1/r$. On the other hand, in cascade theory, interactions are taken into account by cross sections which involve matrix elements squared. There is no room for classical charge coherence.

There are a whole variety of problems one can address in heavy ion collisions such

- What is the equation of state of strongly interacting matter?
- Is there a first order QCD phase transition?

These issues and others would take us beyond the scope of these lectures. The issues which I would like to address are related to the determination of the initial conditions, a problem which can hopefully be addressed using weak coupling methods in QCD.

7. Universality

There are two separate formulations of universality which are important in understanding small x physics.

The first is a weak universality. This is the statement that physics should only depend upon the variable[7]

$$\Lambda^2 = \frac{1}{\pi R^2} \frac{dN}{dy} \tag{17}$$

As discussed above, this universality has immediate experimental consequences which can be directly tested.

The second is a strong universality which is meant in a statistical mechanical sense. At first sight it appears to be a formal idea with little relation to experiment. If it is however true, its consequences are very powerful and far reaching. What we shall mean by strong universality is that the effective action which describes small x distribution function is critical and at a fixed point of some renormalization group. This means that the behavior of correlation functions is given by universal critical exponents, and these universal critical exponents depend only on general properties of the theory such as the symmetries and dimensionality.

Since the correlation functions determine the physics, this statement says that the physics is not determined by the details of the interactions, only by very general properties of the underlying theory!

8. Acknowledgments

This lecture was originally presented in more or less its current form as the first of four lectures at the 40'th Schladming Winter School: DENSE MATTER, March 3-10, 2001.

I thank my colleagues Alejandro Ayala-Mercado, Adrian Dumitru, Elena Ferreiro, Miklos Gyulassy, Edmond Iancu Yuri Kovchegov, Alex Kovner, Jamal Jalilian-Marian, Andrei Leonidov, Raju Venugopalan and Heribert Weigert with whom the ideas presented in this talk were developed. I particularly thank Kazunori Itakura for a careful and critical reading of the manuscript.

I thank the organizers of this meeting, Frithjof Karsch and Helmut Satz, for bringing us all together in the pleasant surroundings of ZiF where over the years, and I hope in the future, so much exciting physics has been done.

This manuscript has been authorized under Contract No. DE-AC02-98H10886 with the U. S. Department of Energy.

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