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Measuring nonlinear momentum compaction in RHIC*

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Abstract

The chromatic nonlinearity parameter, α_1, has a strong impact on longitudinal dynamics in the vicinity of transition [1, 2, 3]. Measurements of the synchrotron frequency as a function of radius are used to constrain the value of α_1.

1 PRELIMINARIES

The lattice parameters α_0 and α_1 relate the change in closed orbit path length C with the reference value for the center of the beam pipe C_0 and the fractional momentum difference between the closed orbit momentum p and the reference value for the center of the beam pipe p_0 via [1, 2, 3]:

$$\frac{C}{C_0} = 1 + \alpha_0\delta\left(1+\alpha_1\delta\right) + O(\delta^3) \tag{1}$$

where $\delta = (p-p_0)/p_0$ is the fractional momentum difference of the closed orbit and reference orbit momenta. To get the revolution frequency one needs the change in velocity (βc) between the closed and reference orbits.

Define $u = p/mc$ then :

$$\begin{aligned} \frac{1}{\beta} &= \left(1+u^{-2}\right)^{1/2} &(2)\\ \frac{d}{du}\frac{1}{\beta} &= -u^{-3}\left(1+u^{-2}\right)^{-1/2} &(3)\\ \frac{d^2}{du^2}\frac{1}{\beta} &= 3u^{-4}\left(1+u^{-2}\right)^{-1/2} \\ &- u^{-6}\left(1+u^{-2}\right)^{-3/2} &(4)\end{aligned}$$

Since $\delta = (u-u_0)/u_0$

$$\frac{1}{\beta} = \frac{1}{\beta_0}\left(1 - \frac{\delta}{\gamma_0^2} + \frac{\delta^2}{2}\left[\frac{3}{\gamma_0^2} - \frac{1}{\gamma_0^4}\right]\right) + O(\delta^3) \tag{5}$$

The revolution period is $T = C/\beta c$ so

$$\begin{aligned} \frac{T}{T_0} &= 1 + \left(\alpha_0 - \frac{1}{\gamma_0^2}\right)\delta \\ &+ \delta^2\left(\alpha_0\alpha_1 - \frac{\alpha_0}{\gamma_0^2} + \frac{3}{2\gamma_0^2} - \frac{1}{2\gamma_0^4}\right) + O(\delta^3) \quad (6)\end{aligned}$$

The data are synchrotron frequency versus radius. These were obtained with the 2GHz Schottky cavity and the values at the peaks in the synchrotron spectrum correspond to small amplitude synchrotron oscillations. Therefore, a linear expansion of the equations of motion about the stable fixed point will suffice. Define

$$\hat{\delta} = \frac{p-p_s}{p_0} = \frac{p-p_0}{p_0} - \frac{p_s-p_0}{p_0} = \delta - \delta_s \tag{7}$$

where p_s is the synchronous momentum, and let $\tau = T - T_s$ be the difference in revolution period between a particle and the synchronous particle. The experiment was done at constant magnetic field below transition so:

$$\begin{aligned} \frac{d\hat{\delta}}{dn} &= \frac{qV}{\gamma_0\beta_0\beta_s mc^2}(\omega_{rf}\tau) &(8)\\ \frac{d\tau}{dn} &= T_0\hat{\delta}\left(\frac{d}{d\delta}\frac{T}{T_0}\right)_{\delta=\delta_s}, &(9)\end{aligned}$$

where n is the turn number. The synchrotron frequency is given by

$$f_s^2 = \frac{qV}{\gamma_0\beta_0 mc^2}\frac{\omega_{rf}^3}{4\pi^2h^2}\frac{T_0}{\beta_s}\left(\frac{d}{d\delta}\frac{T}{T_0}\right)_{\delta=\delta_s}, \tag{10}$$

where $h = 360$ is the harmonic number. In equation (10) only ω_{rf}, β_s and the derivative term vary with radial steering and they are tightly related since

$$\frac{\omega_{rf}}{\omega_{rf,0}} = \frac{T_0}{T_s}. \tag{11}$$

Taking the logarithm of equation (10), differentiating with respect to δ_s, and evaluating at $\delta_s = 0$ yields

$$\begin{aligned} \frac{2}{f_s}\frac{df_s}{d\delta_s} &= 2\frac{\alpha_1 + \frac{3}{2\gamma_0^2\alpha_0} + O(\alpha_0)}{1 - \frac{1}{\gamma_0^2\alpha_0}} \\ &- 3\left(\alpha_0 - 1/\gamma_0^2\right) - \frac{1}{\gamma_0^2}. \quad (12)\end{aligned}$$

In equation (12) the $O(\alpha_0)$ appearing in the numerator of the first term on the right are found in equation (6) and are neglected since they produce a very small correction. Also the second and third terms on the right of equation (12) are very small near transition and will be neglected.

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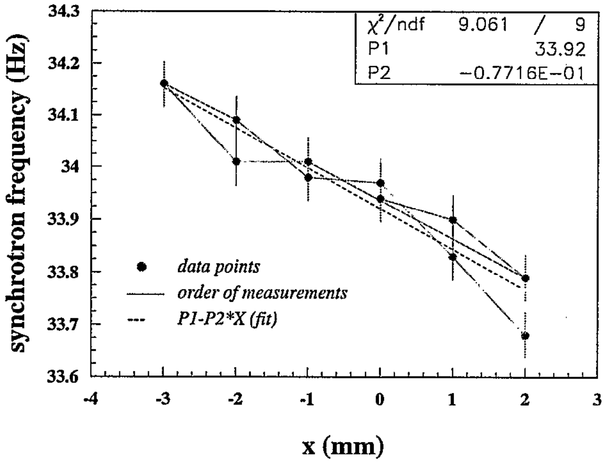

Figure 1: Measured synchrotron frequency versus radial steering setpoint.

2 APPLICATION

The data for the yellow ring and a least squares fit are shown in Figure 1. The first measurement was at $x = 0$ and the second at $x = 1$ mm. Notice that the chronological order of the measurements went to large then small then large radius. There are no systematic drifts. An ideal χ^2 requires 1σ errors of 0.046 Hz. The resolution bandwidth of the spectrum analyzer was 1 Hz and measurements were made using 11 synchrotron lines (10 f_s). The expected error is $\sim \sqrt{2}/(10\sqrt{12})$ Hz $= 0.04$ Hz, where the $\sqrt{12}$ comes from a boxcar distribution. The factor of 10 comes from a span of $10 f_s$ and the additional $\sqrt{2}$ comes from the two independent measurements of synchrotron frequency at the edges of the 11 line span.

The reference value for the energy was $\gamma_0 = 20$ and both the horizontal and vertical chromaticities were $\lesssim 1$. Assume a bare value of $\gamma_t \equiv 1/\sqrt{\alpha_0} = 22.76$. Assuming the frequency steering is accurate at this value of gamma

$$\frac{df_s}{d\delta} = R_0 \alpha_0 \frac{df_s}{dx}, \tag{13}$$

where $R_0 = 610.2$ m is the reference radius and x is the horizontal position. From the fit to the data

$$\frac{1}{f_s}\frac{df_s}{d\delta_s} = -2.68 \pm 0.23 \tag{14}$$

Now since

$$1 - \frac{{\gamma_t}^2}{\gamma_0^2} = -0.295, \qquad \frac{3{\gamma_t}^2}{2\gamma_0^2} = 1.94,$$

one obtains the final result

$$\alpha_1 = -1.15 \pm 0.10.$$

The MAD[4] model of the RHIC ring predicts

$$\alpha_1 = -1.0 \text{ for } Q'_y = Q'_x = 0$$

and

$$\alpha_1 = -1.2 \text{ for } Q'_y = Q'_x = 2.$$

Given natural chromaticities $Q'_x \sim Q'_y \sim -30$, the results are in fair agreement with the model.

3 REFERENCES

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