BNL-66689

Polarized Protons Tracking in the AGS and RHIC [1]

A.U. Luccio Brookhaven National Laboratory, Upton, NY 11973, USA. luccio@bnl.gov

Abstract: A code, SPINK, to track polarized particles in a circular accelerator, in particular RHIC [1], is been used to: • Find conditions for safely crossing depolarizing resonances, using Siberian Snakes. • Find the best conditions to match the spin of the injected beam to the ring lattice. • Study the operation of Spin Rotators. • Study the beam-beam effects in a polarized proton collider.

1 Basic Operation of SPINK

SPINK tracks polarized protons in the 6 dimension phase space of beam dynamics and the 3 dimension space of spin [2]. Beam phase space is the transverse $\vec{r} = (x, p_x, y, p_y)$ phase space, plus the longitudinal $(\phi, dp/p)$. We define $\hat{y}$ as the vertical coordinate, for an horizontal ring. Spin is a real unitary vector $\vec{s} = (S_x, s_y, s_z)$.

Orbit tracking is done using first order matrices and second order "Transport" maps provided by the optical code MAD. In input SPINK accepts Mad CERN Vers.8 or the BNL version [3]. In order to decrease the number of MAD machine modules, all modules that do not contribute to spin precession are lumped together to form passive drifts using the expression

$$R_{ij} = \sum_{n=1}^{6} R''_{ik} R'_{kj} \quad T_{ijk} = \sum_{n=1}^{6} \left(R''_{in} T'_{njk} + \sum_{m=1}^{6} T''_{inm} R'_{nj} R'_{mk} \right) \tag{1}$$

R and T are first and second order orbit Transport maps, respectively, and ('), (") denote the order in which maps are taken.

SPINK tracks one or many representative particles, deposed on the contour of phase space ellipses or randomly extracted inside the phase space. So far, machine modules in SPINK are <u>thick</u> for the orbit and <u>thin</u> for spin rotation, with notable exceptions (snakes and spin rotators). In some cases (say, the AGS with alternating gradient magnets), the thin module model is not accurate and elements are subdivided in slices.

Matrices of thin elements for Spin Rotation by an angle μ around a precession axis $\hat{b}$ are

$$S = I \cos\mu + W \frac{1 - \cos\mu}{\omega^2} + A \frac{\sin\mu}{\omega}, \tag{2}$$

with I the unitary matrix, W a symmetric matrix, and A an anti symmetric matrix

$$W = \begin{pmatrix} b_2^2 & b_1 b_2 & b_1 b_3 \\ b_2 b_1 & b_2^2 & b_2 b_3 \\ b_3 b_1 & b_3 b_2 & b_3^2 \end{pmatrix}, \; A = \begin{pmatrix} 0 & b_3 & -b_2 \\ -b_3 & 0 & b_1 \\ b_2 & -b_1 & 0 \end{pmatrix}, \; \omega^2 = \sum b_i^2 \, . \tag{3}$$

[1] Work Performed under the auspices of the U.S. Department Of Energy

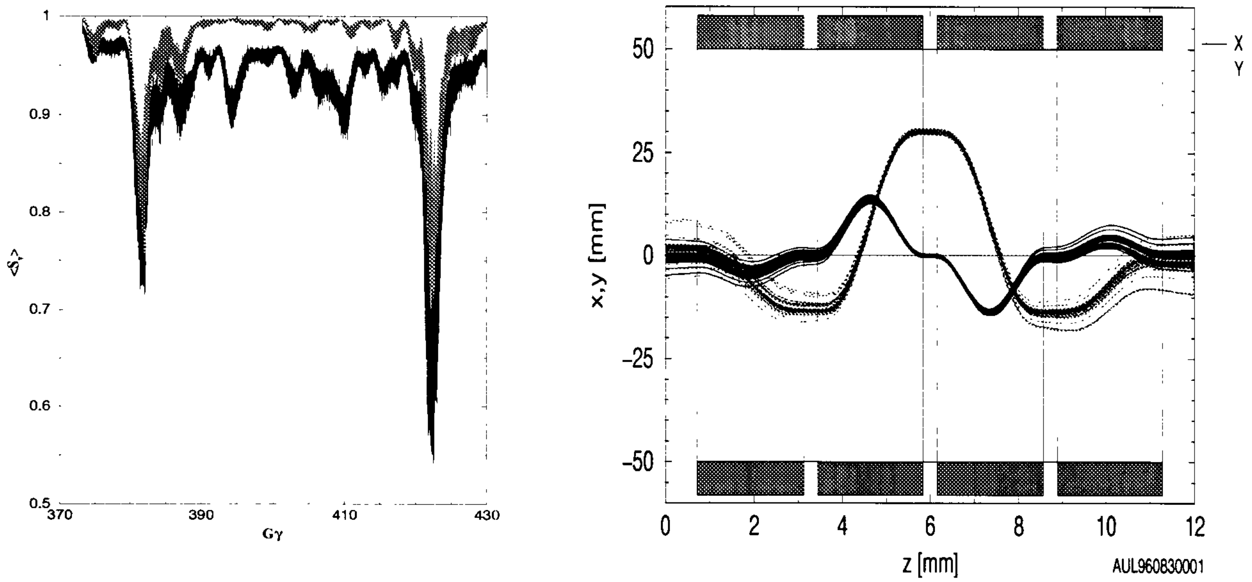

Figure 1: *(a) Spin tracking in RHIC, crossing the* $G\gamma$ = *381.82 and 422.18 intrinsic resonances. The upper curve is for an ideal orbit. The lower for a distorted orbit, due to lattice errors. Average over a few hundred particles tracked. (b) Envelope of trajectories used to calculate R and T orbit and spin maps in a 4-helices Siberian Snake.* $\gamma = 27$, $\epsilon = 20\pi$ *mm-mrad.*

- Orbit Errors

MAD has powerful procedures (Micado) to correct distorted orbits caused by lattice misalignments and field errors. Since the strength of spin resonances depends from the distance of a particle from the equilibrium orbit, depolarization at a resonance is affected by lattice errors. However, the best orbit correction not necessarily provides the best spin resonance correction.

To study the problem, MAD transmits to SPINK the information on the errors (measured or randomly assumed) of each machine element. SPINK accordingly displaces and rotates the corresponding orbit transport maps. Results of proton spin tracking in RHIC for two intrinsic resonances, for an ideal machine, and for errors in the lattice but with a corrected orbit (to 0.2 mm), are shown in Fig.1(a). Here, we assumed random errors of the order of 1 mm and 1 mrad in position and angle of all magnets, respectively.

- Second Order Effects

Second order effects in SPINK are dealt with using the second order Transport maps from MAD. Effects on spin resonances due to crossing of sextupoles and higher order multipoles in the lattice have been clearly show.

Two comments are in order: (i) Second order tracking considerably reduces the speed of calculation. (ii) Second order Transport maps are truncated and therefore non-symplectic. Then, in second order tracking for many turns the emittance may artificialy grow, resulting in wrong answers. Better maps (from Lie Algebra) will be soon implemented.

2 Special Machine Modules

• Snakes and Rotators

In RHIC there are two Snakes and four Rotators per ring, made of four helical dipoles each [4]. SPINK has three ways to deal with helical devices of this kind:

(i) Synthetic Snakes and Rotators, represented by a thin machine module, with unitary orbit matrix, where the components of the spin is rotated by an angle given in as input, around a given axis. Standard RHIC snake axes are at $\pm 45^0$ with respect to the orbit.

(ii) Analytical Snakes. In this case the orbit matrix is derived [5] from a solution of the equation of motion and the BMT equation for spin in each helix in a first order Blewett-Chasman [6] field. Spin rotation is similarly dealt with [7]. For a helix of length L and maximum field B_0 it is

$$\omega = (1+G\gamma)\frac{B_0}{B\rho},\ k = \pm 2\pi/L,\ \mu = -2\pi\sqrt{1+(\omega/k)^2},\ \phi = \tan^{-1}(\omega/k). \tag{4}$$

μ is the spin rotation angle, ϕ the axis angle on the horizontal plane.

(iii) Map-Generated Matrices For Helical Snakes

Transport in helical devices is implemented by symplectic transformations up to third order of the coordinates, calculated by tracking a number of particles in phase space and spin space through magnetic field maps [8]. Transformations coefficients are given as a function of the beam energy. Fig. 1(b) shows the geometry of a RHIC 4-helix Siberian snake for RHIC and the trajectories of protons used to calculate the transport maps by third-order fitting.

• RF Dipoles.

In an oscillating or a rotating magnetic field B at frequency $\omega_R F$ the orbit and the spin receive a kick. The resulting spin precession angle and the angle kick of the orbit(ω_0 is the revolution frequency in the ring) are

$$\mu = G\gamma \int Bd\ell/B\rho,\ \delta p_\perp = \frac{\int Bd\ell}{B\rho}\cos(\nu_m\theta(t)),\ \nu_m = \frac{\omega_{RF}}{\omega_0}. \tag{5}$$

We use RF Dipoles and simulate them in SPINK for two purposes:

(i) As Spin Flippers: The frequency of the field is continuously varied. When it reaches a value equal to half the revolution frequency in RHIC (with two snakes) the spin flips. This is a useful operation when doing experiments with polarized colliding beams.

(ii) To produce an intrinsic Resonance Jump [9]: If an horizontal RF dipole field is applied at a frequency close to the betatron frequency, the beam can be adiabatically moved to a resonant island in phase space. Then, the beam can be accelerated through the resonance by maintaining its integrity. This technique, successfully applied to the AGS, is also used in SPINK to calculate the stable spin axis direction in every point of phase space for a given energy of the beam.

• RF Accelerating Cavities.

Cavities increase the longitudinal component of the beam momentum and reduce the transverse momentum. So, the energy of the particle increases and the transverse emittance is reduced by a factor $1/\beta\gamma$. Cavities are represented in SPINK by matrices and by coupled Courant-Snyder pendulum equations for the synchrotron phase and energy.

- Fine Betatron Tune adjustement.

When the fractional value of the betatron frequency is close to some characteristic number, related to the lattice periodicity (e.g. 1/6 or 5/6 in RHIC), not only the orbit may encounter resonance instabilities, but also a spin resonance can be enhanced. To explore in detail by tracking the forest of resonance lines in the betatron space with a realistic beam, we must gradually change the betatron frequency. This is done in SPINK by padding each quadrupole on both sides with two thin extra quadrupoles of variable strength.

3 Validation

- Validation Vs. Theory.

A simulation code must be validated against known theory. For a spin tracking code at least the following things should happen, and have been successfully checked:

(i) Orbit dynamics: Orbit transfer maps should be symplectic in a way that some dynamical quantities, in particular the beam oscillation normalized amplitudes

$$a = \sqrt{[\xi^2 + (\alpha_T\xi + \beta_T\xi')]\,\beta\gamma} \tag{6}$$

with $\xi = (x \text{ or } y)$ and α_T, β_T Twiss functions, will remain constant during acceleration.

(ii) Distorted orbit reproduced by tracking in SPINK, when matrices are moved and rotated according to the errors transmitted by MAD, should coincide with the distorted orbit calculated in MAD with different techniques.

(iv) Spin precession and resonance crossing: resonance strength ϵ_k, calculated according to the theory, can be directly checked using the Froissart-Stora formula

$$\epsilon_k = \sqrt{-4\alpha \ln\left(\frac{1}{2}(1+ < P >)\right)}\,, \tag{7}$$

with α the rate of resonance crossing ($\Delta\gamma_n$ is the change of γ per turn) and $< P >$ is the final value of the polarization.

- Validation vs. Experiments.

At the present we can only count on experiments performed at the AGS. SPINK has extensively been used to simulate the experiments and to interpret and predict results. Results have been very encouraging [10].

4 Beam Beam

In RHIC, the two accelerated counter rotating beams collide at six locations along the ring. Collisions affect both orbit and spin dynamics, since the electric and magnetic field of a beam exercise forces on the particles of the other beam.

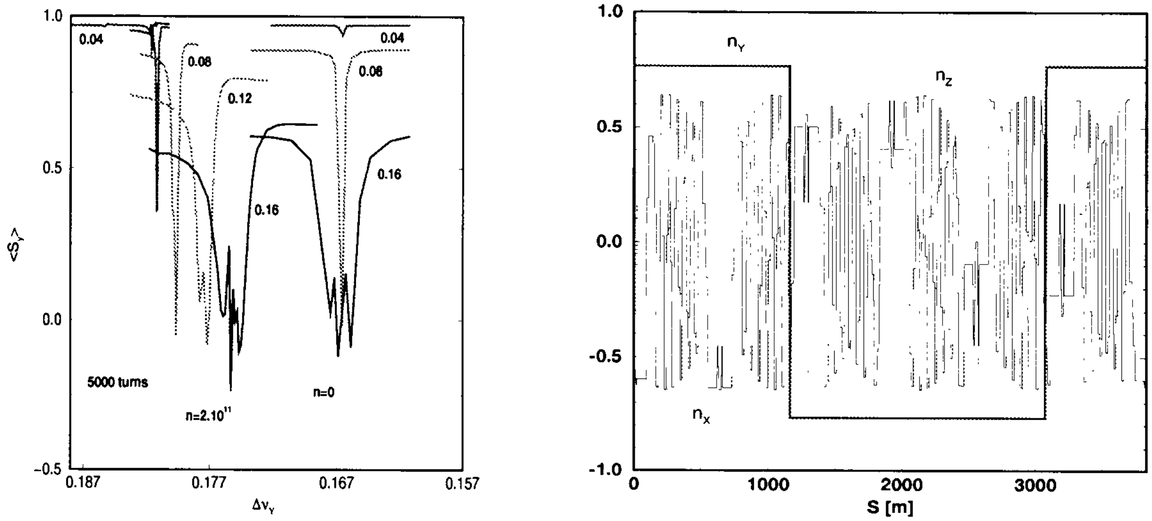

Figure 2: *(a) Vertical component of the spin with Beam-beam in a resonance crossing. The tune of particles at various vertical distances from axis determines the resonance strength. No space charge, and* $n = 2 \times 10^{11}$. *(b) stable spin axis for spin rotation* 180^0 *and* 100^0 *in two snakes around different axis settings.*

In SPINK, we modeled one beam as a Gaussian distribution of charges and studied the effect of the fields on the single particles of the other beam [11]. The resulting betatron tune shift in a collision is

$$\Delta\nu = -\frac{\xi}{u^2}\left[1 - \exp -u^2 I_0(u^2)\right], \tag{8}$$

where $u = r/2\sigma$ and ξ is the tune shift on axis, proportional to the number of particles per bunch n and inversely to the normalized emittance. For $n = 2 \times 10^{11}$, $\epsilon = 10\pi$ mm-mrad and $\beta^* = 1m$, it is on axis $\xi = -0.015$. Tune distribution in the beam can be calculated by the equation above, in good agreement with SPINK tracking.

Fig. 2(a) shows how the vertical component of the spin is affected by beam-beam at fixed energy corresponding to $G\gamma = 261$, close to the strong intrinsic spin resonance at $G\gamma = 232+\nu_y$.

5 Spin Field

For each position along the accelerator lattice and for each position in phase space, there is a stable spin axis. If the spin of a proton is initially aligned along this direction, it will remain stable, if it isn't, it will precede in a cone around this direction. Finding the stable spin cone corresponding to a given phase space distribution of the particles in the beam is particularly important at injection, since it should coincide with the spin cone of the injected beam.

- Spin Tune.

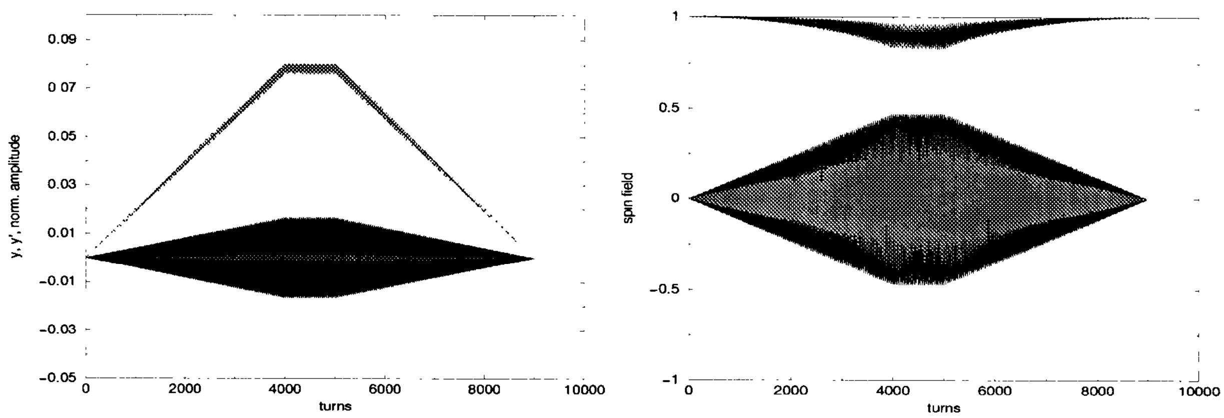

Figure 3: *(a) Orbit brought out of the median plane and back by using an RF dipole. (b) Spin components following the orbit, at injection in RHIC:* $G\gamma = 41.5$. *Upper curve:* S_y. *Lower curves:* S_x *and* S_z.

Spin tune for a polarized particle is the fractional number of spin revolutions per turn in an accelerator. It is measured by the quantity $G\gamma$, and increases with the energy of the beam. When, during the acceleration cycle, the spin tune crosses some specific values, spin-depolarizing resonances appear. Siberian snakes in the lattice make the spin tune independent of energy. For RHIC, with two Siberian snakes, the spin tune is made equal to one. This means that the spin pattern should remain identical and repeat at each turn.

- Stable Spin Axis.

To calculate the stable spin axis direction on the equilibrium orbit , SPINK uses the method of Stroboscopic Average [12]. I.e. a number of protons with different initial spin directions are tracked for a number of turns, forward and backward, until they return to the initial location along the ring. The spin oscillations are then averaged.

Fig.2(b) shows an example of calculated stable spin in RHIC on axis, for $G\gamma = 41.5$. Here, the two snakes were set for spin rotation of 180^0 and 100^0, respectively (producing a stable spin axis on a cone of semi-aperture $\theta = 40^0$). Each snake axis orientation produces an angle ϕ for the stable spin axis. The vertical component of the spin axis is flipped at the two locations of the snakes.

To calculate with SPINK the Spin Field everywere in the transverse phase space, we start a particle on the equilibrium orbit, and find the stable spin axis there with the method described above. Then, we move adiabatically the working point in the transverse phase space with a RF Dipole to larger amplitudes. The particle is finally brought back on the equilibrium orbit, Fig.3(a). The spin follows the evolution of the precession axis, painting a complete picture the spin field [13]. As Fig. 3(b) shows, the three components of the spin also adiabatically follow the orbit bump induced by the RF field.

6 Acknowledgments

Many have contributed to the development of the tracking code SPINK. Let us mention in particular: M. Bai, H. Huang, A. Lehrach, T. Roser, M. Syphers, N. Tsoupas, BNL; T. Katayama, H. Wu, M. Xiao, RIKEN; M.Conte and M.Palazzi, Univ. of Genova, Italy.

References

[1] T.Roser "The Acceleration of Polarized Protons to High Energies", PAC99, New York, NY 1999.

[2] A.U.Luccio in "Trends in Collider Spin Physics", Trieste, Italy, 5-8 Dec. 1995, p.235. • A.U.Luccio, T.Katayama and H. Wu, PAC97, Vancouver, BC, Canada, 1997, p.92.

[3] H.Grote and F.C.Iselin, "The MAD Program Vers. 8.16" CERN/SL/90-13, Geneva, Switzerland, May, p 26, 1995. • J.Niederer, "Z Twiss Tracking Matrix File" BNL MAD Programming Note, Upton, NY, updated March 1999.

[4] V.I.Ptitsin and Yu.M.Shatunov, in "Proc Third Workshop on Siberian Snakes and Spin Rotators", BNL-52453, Upton, NY, Sept. 12-13, 1994, p. 15.

[5] E.D.Courant, AGS/RHIC/SN 004, Upton, NY, Nov 8, 1994.

[6] J.P. Blewett and R.Chasman, J. Ap. Phys 48 (1977) 2692.

[7] M.J.Syphers AGS//RHIC/SN 020, Upton, NY, Feb 1996.

[8] A.Luccio Riken AF-NP-235, Wako, Japan, October 1996. M.Xiao, E.Forest and T.Katayama, CNS-REP-14, Tokyo, Oct. 1998.

[9] M. Bai et al. Proc PAC97, Vancouver, BC, Canada 1997.

[10] H.Huang, T.Roser, A.Luccio, AGS/RHIC/SN 043, Nov 6, 1996.

[11] A.U.Luccio and M.Syphers AGS/RHIC/SN No.068, Upton, NY, Nov. 25, 1997.

[12] K.Heinemann and Hoffstätter, Phys.Rev. E54, 4240, 1995. • D.P.Barber and M.Vogt, EPAC'98, Stockholm, Sweden, 1998, p. 1362.

[13] A.Lehrach. This Workshop