

## SIMULATION STUDY OF THREE-DIMENSIONAL LASER COOLING SCHEMES FOR FAST STORED BEAMS

T. Kihara, H. Okamoto, Y. Iwashita,

NSRF, Institute for Chemical Research, Kyoto University, Kyoto, Japan

K. Oide, KEK, Tsukuba, Japan

G. Lamanna, Dipartimento di Fisica dell'Universita and INFN, Bari, Italy

J. Wei,<sup>\*</sup>BNL, New York, USA

### Abstract

Molecular dynamics (MD) approach is employed to study laser cooling of fast circulating beams in a storage ring. We compare several three-dimensional (3D) cooling methods, examining achievable minimum beam temperature. In particular, the stress is put upon the three coupling schemes, i.e. the dispersion-coupling scheme, the coupling-cavity scheme, and the tapered cooling scheme. We show that beam temperatures much lower than the currently achievable level could be reached with these schemes.

### 1 INTRODUCTION

Laser cooling has been well-known as a powerful tool to attain ion beams with extremely high phase-space densities [1]. It has already been experimentally demonstrated that one can produce an ultra-cold beam close to the longitudinal space-charge limit [2, 3]. Direct transverse laser cooling of fast circulating beams is, however, thought to be quite difficult due to the poor interaction of a transverse laser with stored ions. Although the sympathetic effect naturally caused by Coulomb interactions between individual ions can be a source of heat transfer, the equilibrium temperatures of the horizontal and vertical directions are still much higher than that of the longitudinal direction according to the past experimental observations [4].

To overcome this difficulty, a novel method has been proposed in the previous publications [5, 6]. The idea is simple, that is, we take advantage of a synchrotron coupling to indirectly increase the transverse cooling rates. To incorporate the idea in a real storage ring, two analogous schemes, i.e. the coupling-cavity scheme and the dispersion-coupling scheme, have been explored both analytically and numerically. It has been proven that the transverse cooling rates can be most enhanced under the resonance conditions

$$\nu_x - \nu_z \approx \text{integer} \text{ and } \nu_x - \nu_y \approx \text{integer}, \quad (1)$$

where  $\nu_x$ ,  $\nu_y$ , and  $\nu_z$  are, respectively, the horizontal, vertical and longitudinal tunes.

<sup>\*</sup>Work performed under the auspices  
 of the U.S. Department of Energy.

In addition to these two coupling schemes, an even more promising scheme, called "tapered cooling", has recently been studied in detail [7]. Mathematically, the taper cooling can be modeled as

$$\Delta \left( \frac{\delta p}{p} \right) = -f_s \left[ \left( \frac{\delta p}{p} \right) - \gamma C_x \frac{x}{\rho_m} \right] \quad (2)$$

where  $\delta p/p$  and  $x$  are the momentum deviation and horizontal displacement in the laboratory frame,  $\rho_m$  is the average radius of curvature in the bending sections of the storage ring,  $\gamma$  is the Lorentz factor,  $f_s$  is a positive constant corresponding to the cooling strength,  $\Delta(\delta p/p)$  stands for the change in  $\delta p/p$  before and after the laser cooling section, and we call  $C_x$  *tapering factor*. This cooling scheme is particularly important for realizing beam crystallization since the basic characteristic of the tapered force just matches the nature of crystalline ground states. Furthermore, it works not only for bunched beams but also for coasting beams. As shown below, tapered cooling enables one to attain all kinds of crystalline structures.

### 2 THE RESONANT COUPLING SCHEMES

In this section, we investigate, by using a MD code, how much one can expect from the dispersion-coupling scheme and the coupling-cavity scheme. There presently exist only two storage rings in which a laser cooling system has been installed; i.e. the TSR ring in Germany [2] and the ASTRID ring in Denmark [3]. However, neither of these rings satisfy the necessary conditions of beam crystallization [7], so we consider, among a wide range of choice, the lattice parameters of the storage ring TARN II [8] in the following numerical simulations. The main parameters assumed here are summarized in Table 1.

First of all, we need to adjust the betatron tunes of TARN II such that the resonance conditions in Eq. (1) are approximately satisfied. Apparently, the non-integer parts of  $\nu_x$  and  $\nu_y$  must be as small as possible in order to guarantee that the RF voltage applied to stored ions is within a reasonable range. To avoid the excitation of integer resonance in the transverse motions, we here set  $\nu_x$

and  $\nu_y$  to be 2.11 and 1.08 respectively. Eq. (1) then indicates that the optimum synchrotron tune is around 0.1. Since the kinetic energies of laser-cooled beams are usually rather low, it is not difficult to realize this high  $\nu_y$  with a sufficiently low RF voltage. For instance, the RF voltage necessary to achieve  $\nu_y=0.1$  for a 1 MeV  $^{24}\text{Mg}^+$  ion

Table 1: Parameters of TARN II

Ring circumference	77.7 m
Betatron tunes, $\nu_x, \nu_y$	2.11, 1.08
Dipole bending radius, $\rho_m$	4.01 m
Skew quad strength	0.026 m <sup>-1</sup>
Ion species	$^{24}\text{Mg}^+$
Kinetic energy	1 MeV
RF harmonic number	1000
RF voltage of the regular cavity	< 150 V
Coupling strength of the coupling cavity (See Ref. [5])	0.001

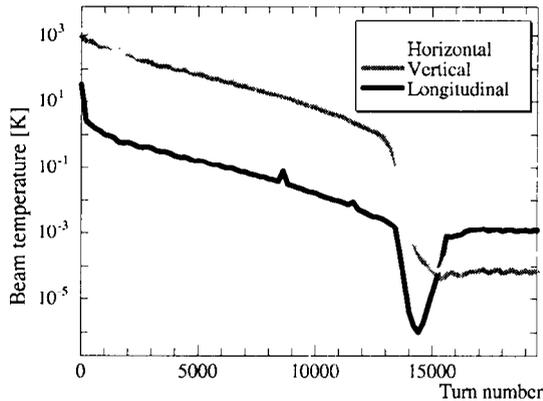


Figure 1: Molecular dynamics result on the dispersion-coupling scheme. The coupling cavity has been switched off, and  $\nu_y=0.05$ .

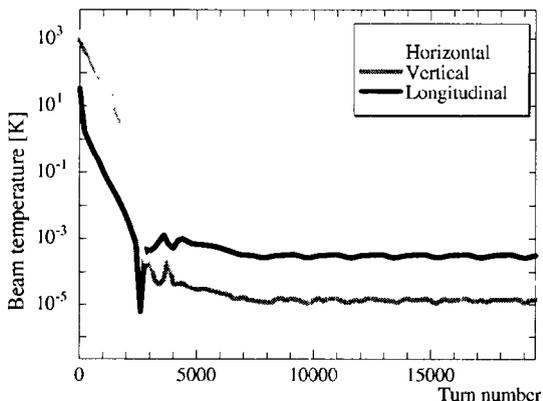


Figure 2: Molecular dynamics result on the coupling-cavity scheme.  $\nu_y=0.05$ .

is only about 150 V when the harmonic number of 1000 is chosen.

A typical MD result for the TARN II lattice has been given in Fig. 1. In this simulation, 100 ions have been trapped in each RF bucket, so the total number of stored ions are  $10^5$ . We see that the transverse temperatures quickly decrease to around the mK range, although the synchrotron tune has been set somewhat lower than the optimum value to diminish the operating RF voltage. If a coupling cavity is turned on, we can even more enhance the transverse cooling rates, as plotted in Fig. 2. Figure 3 illustrates the real-space profiles of a  $^{24}\text{Mg}^+$  bunch corresponding to the final state of the case in Fig. 2. Clearly, an ordered structure has been formed.

Figure 4 demonstrates the  $\nu_y$ -dependence of equilibrium temperatures when only dispersion coupling is activated. It is interesting to observe that final temperature is even lower at a smaller value of  $\nu_y$ . Noting that we certainly need a much longer time to reach the transverse equilibrium as  $\nu_y$  gets smaller, it may be concluded that a higher cooling rate does not always lead to a lower final temperature. As a matter of fact, we obtain roughly the same result as shown in Fig. 4 even if the coupling cavity is excited.

For the ASTRID lattice, the situation becomes different. Unlike the TARN II case, it is often quite troublesome to stably capture stored ions in an RF bucket. In fact, the time evolution of the beam temperature exhibits a spiky behavior since some ions occasionally escape from the bucket. Furthermore, even with the coupling cavity turned on, the final beam temperature is beyond the 1 K level, at least two orders of magnitude higher than the TARN II case. These substantial differences between the TARN II and ASTRID results could be understood by simply considering the transverse beam stability. As self-consistently proven in Ref. [9], space-charge-dominated beam in a storage ring is trapped by a half-integer Mathieu stopband unless the lattice design satisfies the so-called *maintenance condition* [10]. Since the ASTRID lattice simply breaks this condition, the transverse beam motion cannot be stable in an ultra-cold state, resulting in a high final temperature.

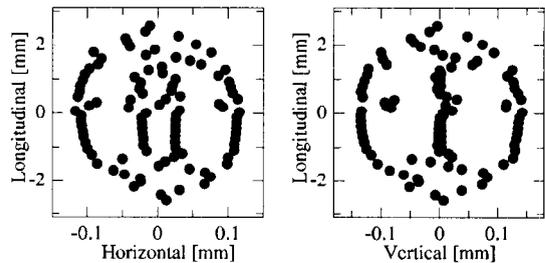


Figure 3: Real-space configurations of the laser-cooled  $^{24}\text{Mg}^+$  beam corresponding to the final state of the case in Fig. 2.

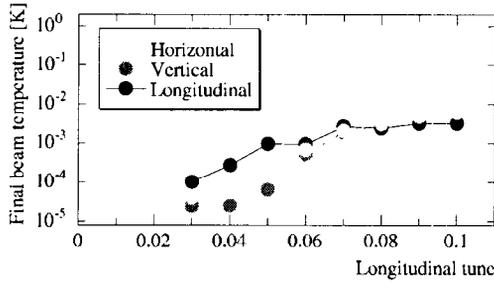


Figure 4: Equilibrium temperature vs.  $\nu_z$ . The total number of stored particles considered in this result is  $10^5$ . While the final temperature tends to be lowered at a smaller value of  $\nu_z$ , the cooling time becomes much longer as  $\nu_z$  is reduced from the resonant value, i.e.  $\nu_z=0.1$  in the present case. Note further that the equilibrium temperature is simply a measure of the distance to a ground state. Ground-state structures can be different under different types of cooling force.

### 3 THE TAPERED COOLING SCHEME

In order to realize the highest possible phase-space density, it is essential to let a cooling force possess the property suitable for crystalline ground states where the revolution frequencies of all stored particles are identical. Specifically, the tapered cooling force expressed by Eq. (2) is appropriate for this purpose [7, 10]. To perfectly stabilize a ground-state structure, the value of the tapering factor must be well optimized. Figure 5 shows a typical dependence of equilibrium beam temperature on the tapering factor. In this MD result, we have again assumed 1 MeV  $^{24}\text{Mg}^+$  ions stored in TARN II. The betatron tunes are  $\nu_x=1.68$  and  $\nu_y=1.85$ . Note further that the beam, consisting of  $3.38 \times 10^6$  ions, is now continuous. We find that the final temperature becomes particularly low in the region around  $C_{xs}=0.267$  where the beam has been completely crystallized. The acceptable range of  $C_{xs}$  for beam crystallization obtained from MD simulations has been plotted in Fig. 6. In order to analytically predict the optimum size of the tapering factor for a specific ring lattice, the theory described in Ref. [11] can be employed.

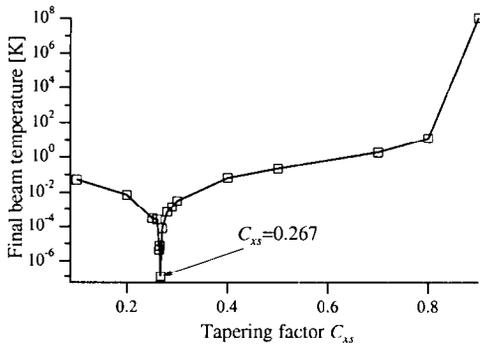


Figure 5: Equilibrium total beam temperature ( $T_x+T_y+T_z$ ) vs. the tapering factor  $C_{xs}$ .

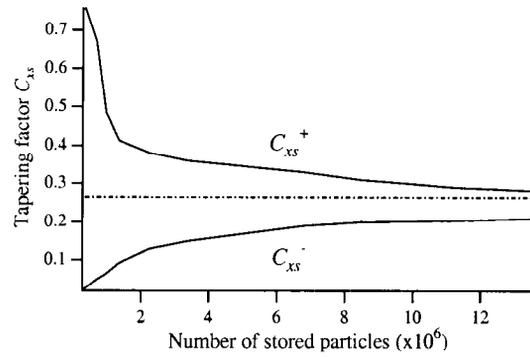


Figure 6: Acceptable range of tapering factor.  $C_{xs}^+$  and  $C_{xs}^-$  represent, respectively, the upper and lower limit of  $C_{xs}$  required for beam crystallization. The dot-dash line indicates the ideal value predicted by the theory in Ref. [11]. The TARN II lattice considered in Fig. 5 has been employed again.

### 4 CONCLUSIONS

We have studied the fundamental properties of the 3D laser cooling methods based on linear synchrotron coupling. It has been clearly demonstrated, through MD simulations, that the transverse cooling time can remarkably be shortened by using the coupling schemes. Provided that the lattice parameters of a ring satisfy the maintenance condition, equilibrium transverse temperature well below 1 K could be reached very quickly. However, to improve the stability of an ultra-cold beam, it eventually becomes necessary to apply the optimized tapered cooling force.

The simulation program used here was originally developed by X.-P. Li and J. Wei. Computation time was provided by the Supercomputer Laboratory, Institute for Chemical Research, Kyoto University.

### REFERENCES

- [1] D. J. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. **20** (1975) 637 ; T. Hänsch and A. Schawlow, Opt. Commun. **13** (1975) 68.
- [2] S. Schröder et al., Phys. Rev. Lett. **64** (1990) 2901.
- [3] J. S. Hangst et al., Phys. Rev. Lett. **67** (1991) 1238.
- [4] H.-J. Miesner et al., Phys. Rev. Lett. **77** (1996) 623.
- [5] H. Okamoto, A. M. Sessler and D. Möhl, Phys. Rev. Lett. **72** (1994) 3977.
- [6] H. Okamoto, Phys. Rev. **E50** (1994) 4982.
- [7] J. Wei, H. Okamoto, and A. M. Sessler, Phys. Rev. Lett. **80** (1998) 2606.
- [8] T. Katayama et al., Proc. of EPAC90, (1990) 577.
- [9] S. Y. Lee and H. Okamoto, Phys. Rev. Lett. **80** (1998) 5133.
- [10] See, e.g., J. Wei, A. Draeseke, A. M. Sessler, and X.-P. Li, BNL Report BNL-52493 (1995).
- [11] H. Okamoto and J. Wei, BNL Report BNL-52546 (1998).