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Longitudinal Space Charge Effect for SNS*

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Introduction

One of performance requirements of the Spallation Neutron Source (SNS) is to keep the uncontrolled beam loss in the storage ring to less than 2×10^{-4} per pulse. For 2 MW SNS, the maximum beam intensity is $N=2\times10^{14}$ protons per ring. Since the bunch lengthening has impact on both the extraction beam loss and the lowering of e-p instability threshold, the longitudinal space charge effect requires attentions. Such a space charge effect has been studied both analytically and using computer simulations.

The longitudinal space charge effect, which is a defocusing force below transition, is a plausible source of the bunch leakage. In this article, the total RF potential, which takes into account the space charge effect together with the RF power, is used to provide analytical predictions for the bunch lengthening. The prediction is confirmed by the computer simulation.

It is found that for 2 MW SNS storage ring, the longitudinal space charge induced bunch leakage into the interbunch gap is not significant. Therefore, corrections to the longitudinal space charge impedance, such as the proposed ferrite insertion in the PSR ring, are probably not necessary.

Applying an RF voltage ramping from 20 *KV* to 40 *KV* during the multiturn injection can further cut the bunch leakage to a negligible degree.

The same approach applied to the PSR shows that the longitudinal space charge effect does cause sizable bunch leakage at the intensity limit encountered there.

Space Charge

As usual, the longitudinal space charge impedance is defined as,

$$Z_{SC} = -j\frac{Z_0}{2\beta\gamma^2} \qquad (1)$$

where $Z_0=377\Omega$ and

$$g=1+2\log\left(\frac{b}{a}\right) \qquad (2)$$

with a and b being average radii of the beam an vacuum chamber, respectively. Note that we use $a=\sqrt{2}\sigma$, where vacuum chamber radius b=10 *cm* and the beam size a=2.36 *cm* give rise to the longitudinal space charge impedance $Z_{SC}=-j196\Omega$.

For a beam current I, the space charge impedance induced voltage, a particle sees per turn, can be written as,

$$V_{SC} = \frac{dI}{d\phi}|Z_{SC}| \qquad (3)$$

where ϕ is the beam phase deviation in radius. This voltage represents a longitudinal defocusing force at the SNS, reducing the effective RF voltage within the bunch passage.

We first consider an analytical approach to the longitudinal space charge effect for the SNS. The space charge potential is defined using V_{SC} as,

$$U_{SC}(\phi_\ell) = \int_0^{\phi_\ell} V_{SC}d\phi = \int_0^{\phi_\ell}\frac{dI}{d\phi}|Z_{SC}|d\phi \qquad (4)$$

where ϕ_ℓ is the half bunch length in radius. The RF voltage potential (with second RF) at the bunch length ϕ_ℓ is,

$$U_{RF}(\phi_\ell)=\int_0^{\phi_\ell} V(\sin\phi-0.5\sin2\phi)d\phi \qquad (5)$$

Whether the bunch leaks into the interbunch gap or not is determined by two factors. The first factor is the particle maximum momentum deviation from the equilibrium momentum, i.e. dp/p. This is mainly defined during the multiturn injection. To find out the beam dp/p in the ring, the Linac beam momentum spread, the RF voltage manipulation, and the space charge effect all have to be considered.

The second factor in determining the bunch leakage is the effective total RF potential at the bunch edge, which is,

$$U(\phi_\ell) = U_{RF}(\phi_\ell) - U_{SC}(\phi_\ell) \qquad (6)$$

For a normal bunch line density, say an azimuthally centered parabolic line density, the space charge potential offsets the RF potential in the bunch passage.

Using the Hamiltonian,

$$H = \frac{|\eta|\omega_{RF}^2}{2\beta^2 E}\left(\frac{\Delta E}{\omega_0}\right)^2 - \frac{e}{2\pi}U(\phi) \qquad (7)$$

one obtains the relation between the total potential (combined with the RF and space charge) and the energy deviation. The corresponding momentum deviation is,

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$$\frac{dp}{p} = \sqrt{\frac{eU(\phi)}{\pi\beta^2 E|\eta|}} \qquad (8)$$

If the maximum particle momentum deviation is larger than the one obtained using (8) at the bunch edge ϕ_ℓ, then a bunch leakage will happen.

An important mechanism of the space charge induced bunch lengthening is the self-regulation. For the same intensity, a longer bunch will carry a smaller space charge potential at the bunch edge. Without loss of generality, we calculate the space charge potential using a cosine bunch shape.

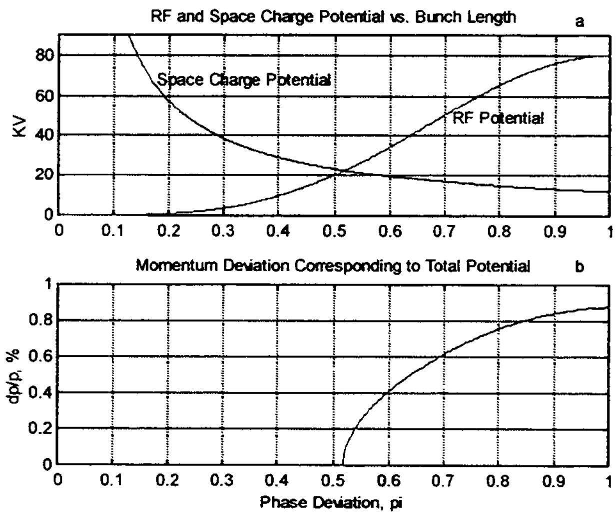

Fig.1. *RF and Space Charge Potential*

In Fig.1a, the space charge potential is compared with the RF potential, under the condition of $N=2\times10^{14}$ and $Z_{SC} = -j196\Omega$. If the half bunch length is 0.52π, then the space charge almost completely offsets the RF potential, the bunch leakage will definitely happen. The total RF potential, however, increases rapidly for the longer bunch, due to both the RF potential increase and the space charge potential decrease. In Fig.1b, the momentum deviation corresponding to the total RF potential is shown. At the half bunch length of 0.69π, the particles with the maximum momentum deviation smaller than 0.6% will not leak into the interbunch gap.

In the case that particle maximum momentum deviation is larger, then the bunch leak will happen. Once the bunch lengthening occurs, the longer bunch reduces the space charge potential at the extended bunch edge. Ideally, this has a negative impact on further bunch lengthening.

Without the space charge effect, the beam maximum momentum spread can be estimated using the Hamiltonian (7). Since the space charge effect would reduce the total beam momentum spread in the injection, simulation is, therefore, needed to find out exact beam momentum spread in the ring.

Simulation of SNS

For SNS, the incoming Linac beam is chopped at the phase deviation $\pm0.67\pi$. The RF voltage (h=1) can be $V_{RF} = 40KV$ per turn. The injection takes about 1,200 turns to get $N = 2\times10^{14}$ protons per pulse.

For the particle longitudinal motion, the following equations are used in the simulation,

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_0}\right) = \frac{e}{2\pi}\left(V_{RF}(\sin\phi - 0.5\sin 2\phi) + V_{SC}\right) \qquad (9)$$

and

$$\frac{d}{dt}\phi = \frac{\omega_0^2\eta}{\beta^2 E}\left(\frac{\Delta E}{\omega_0}\right) \qquad (10)$$

where ΔE is the energy gain per turn. For convenience, the particle momentum deviation dp/p is used in the simulation, with the following relation to the energy deviation,

$$\frac{dp}{p} = \frac{\omega_0}{\beta^2 E}\left(\frac{\Delta E}{\omega_0}\right) \qquad (11)$$

In the simulation, homogeneously distributed Linac beam with momentum spread dp/p=±0.2% is used. In Fig.2a, the particle distributions with the space charge effect is shown, the bunch half length is 0.72π.

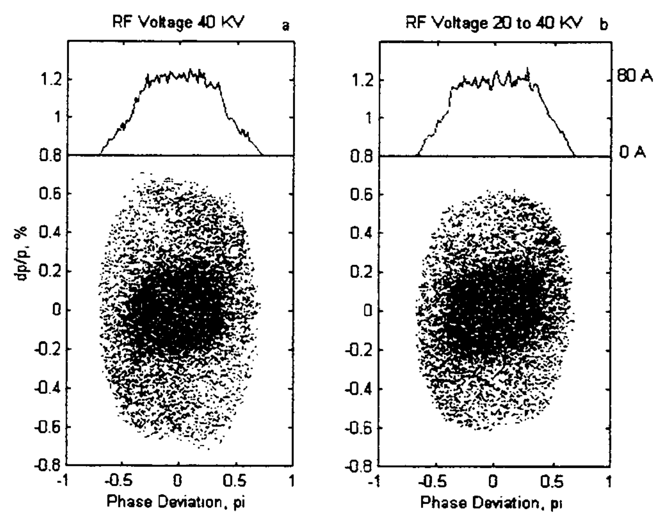

Fig.2. *SNS Space Charge Effect*

To let the RF voltage to ramp from 20 KV at the start to 40 KV at the end will define a smaller beam momentum spread in the ring. Fig.2b shows that the bunch half length stays at 0.67π and the maximum momentum spread is not larger than 0.6%. This agrees with the analytical prediction, which shows that with $V_{RF} = 40KV$, and $N=2\times10^{14}$, half bunch length of 0.69π can contain particles with the maximum momentum deviation smaller than 0.6%. One may observe that the maximum dp/p is just a little larger than that.

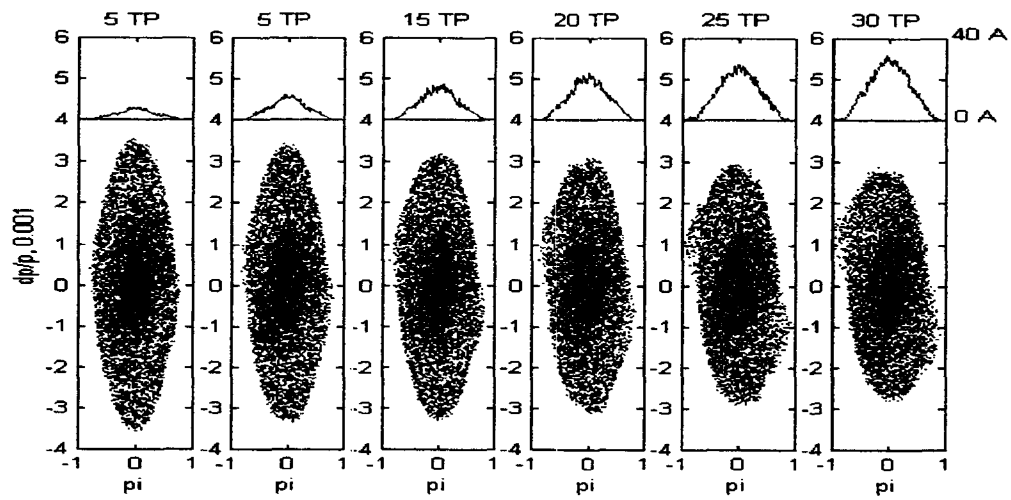

Fig.3. *PSR Space Charge Effect*

Note that the synchrotron frequency is about 1 *KHz* at $V_{RF} = 40KV$, therefore, the entire injection process is less than one synchrotron period. This also contributes to a limited bunch length.

For longitudinal microwave instability, under the condition that the peak current 80 *A*, Keil-Schnell stability criterion requires that $dp/p \geq 0.7\%$. Consider that SNS is operated below transition, $dp/p \approx 0.6\%$ is probably not a problem.

Simulation of PSR

To compare with the situation at the PSR of LANL, the same simulation scheme is applied. The bunch lengthening due to the longitudinal space charge is shown in Fig.3 for different beam intensities. The RF voltage is 6 *KV* and the Linac beam momentum spread is $dp/p = \pm 0.1\%$, chopped at the phase deviation $\pm 0.7\pi$. It can be noticed that the space charge induced bunch leakage becomes significant at $N = 3 \times 10^{13}$, causing a bunch half length 0.96π. Meanwhile, the beam momentum spread in the ring constantly decreases along with the increase of the intensity, presumably due to the longitudinal space charge effect.

For the best possible injection scheme at the PSR, the RF voltage ramping from 6 *KV* at the start to 10 *KV* at the end is utilized to give rise to bunch half length of 0.74π, obtained using the same simulation. This leakage is sizable, but is not strong enough to stimulate the e-p instability. In [1], a possible beam loading effect is speculated as an even stronger mechanism of the bunch leakage.

To combat longitudinal space charge effect, a correction scheme by inserting ferrite in the ring is studied at the PSR [2]. An improvement of the bunch leakage was observed. At the SNS, it seems that based on the analytical and simulation results, the bunch leakage due to the longitudinal space charge is limited. Therefore, a similar correction is probably not necessary.

Discussion

Since at SNS, the extraction beam loss has to be less than 10^{-4}, the longitudinal space charge induced bunch lengthening is an important issue. The longitudinal space charge impedance (determined by factors *a*, *b*, and $\beta\gamma^2$, and $dI/d\phi$) give rise to a space charge potential (4). Therefore, a proper RF potential (5) is needed to combat the space charge bunch lengthening.

1. In a storage ring, the RF accelerating power is not needed. However, a relatively high RF voltage is necessary to confine the bunch length. For SNS, this voltage is between 30 *KV* to 40 *KV*.
2. High intensity beam effectively reduces the beam momentum spread in ring. For instance, with the space charge effect, beam momentum spread at the SNS is 0.6%, rather than 0.8% without it.
3. Second harmonic RF is preferred, not only because of the associated low beam peak current, but also because of its sharp potential at the bunch edge.
4. RF manipulation is important. For instance, the ramping of the RF voltage during the multiturn injection effectively reduces the beam momentum spread defined in ring, which is favorable in bunch length control.
5. Finally, bunch shape formation, affecting $dI/d\phi$, must be considered. It seems that a relatively large momentum spread of the Linac beam, and also proper manipulation on the RF can satisfy this requirement.

Reference

1. S.Y. Zhang and W.T. Weng., ' Beam Loading Effect in Bunch Leakage at the PSR,' in press, NIM.
2. J.E. Griffin et. al., ' Experimental Study of Passive Compensation of Space Charge Potential Well Distortion at the PSR,' Fermilab, FN-661, Nov. 1997.